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On the Fučik spectrum for the p-Laplacian with Robin boundary condition

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ABSTRACT

The aim of this paper is to study the Fučik spectrum of the p-Laplacian with Robin boundary condition given by

$$\begin{aligned} -\Delta_p u &= a(u^+)^{p-1} - b(u^-)^{p-1} && \text{in } \Omega, \\ |\nabla u|^{p-2}\frac{\partial u}{\partial \nu} &= -\beta |u|^{p-2}u && \text{on } \partial\Omega, \end{aligned}$$

where $\beta \geq 0$. If $\beta = 0$, it reduces to the Fučik spectrum of the negative Neumann p-Laplacian. The existence of a first nontrivial curve $\mathcal{C}$ of this spectrum is shown and we prove some properties of this curve, e.g., $\mathcal{C}$ is Lipschitz continuous, decreasing and has a certain asymptotic behavior. A variational characterization of the second eigenvalue λ_2 of the Robin eigenvalue problem involving the p-Laplacian is also obtained.

1. Introduction

The Fučik spectrum of the negative p-Laplacian with a Robin boundary condition is defined as the set $\widehat{\Sigma}_p$ of $(a, b) \in \mathbb{R}^2$ such that

$$\begin{aligned} -\Delta_p u &= a(u^+)^{p-1} - b(u^-)^{p-1} && \text{in } \Omega, \\ |\nabla u|^{p-2}\frac{\partial u}{\partial \nu} &= -\beta |u|^{p-2}u && \text{on } \partial\Omega, \end{aligned} \tag{1.1}$$

has a nontrivial solution. Here the domain $\Omega \subset \mathbb{R}^N$ is supposed to be bounded with a smooth boundary $\partial\Omega$. The notation $-\Delta_p u$ stands for the negative p-Laplacian of u, i.e., $-\Delta_p u = -\text{div}(|\nabla u|^{p-2}\nabla u)$, with $1 < p < +\infty$, while $\frac{\partial u}{\partial \nu}$ denotes the outer normal derivative of u and β is a parameter belonging to $[0, +\infty)$. We also denote $u^{\pm} = \max\{\pm u, 0\}$. For $\beta = 0$, (1.1) becomes the Fučik spectrum of the negative Neumann p-Laplacian. Let us recall that $u \in W^{1,p}(\Omega)$ is a (weak) solution of (1.1) if

$$\int_\Omega |\nabla u|^{p-2}\nabla u \cdot \nabla v \mathrm{d}x + \beta \int_{\partial\Omega} |u|^{p-2}uv\mathrm{d}\sigma = \int_\Omega (a(u^+)^{p-1} - b(u^-)^{p-1})v\mathrm{d}x, \quad \forall v \in W^{1,p}(\Omega). \tag{1.2}$$

If $a = b = \lambda$, problem (1.1) reduces to

$$\begin{aligned} -\Delta_p u &= \lambda |u|^{p-2}u && \text{in } \Omega, \\ |\nabla u|^{p-2}\frac{\partial u}{\partial \nu} &= -\beta |u|^{p-2}u && \text{on } \partial\Omega, \end{aligned} \tag{1.3}$$

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