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Second-order optimality conditions using approximations for nonsmooth vector optimization problems under inclusion constraints[☆]

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ABSTRACT

Second-order necessary conditions and sufficient conditions for optimality in nonsmooth vector optimization problems with inclusion constraints are established. We use approximations as generalized derivatives and avoid even continuity assumptions. Convexity conditions are not imposed explicitly. Not all approximations in use are required to be bounded. The results improve or include several recent existing ones. Examples are provided to show that our theorems are easily applied in situations where several known results do not work.

1. Introduction

Second-order optimality conditions are of great interest, since they refine first-order conditions with second-order information which is much helpful to recognize optimal solutions as well as to design numerical algorithms for computing them. In nonsmooth optimization, similarly as for first-order conditions, the major approach to second-order ones is using generalized derivatives to replace Fréchet and Gateaux derivatives which do not exist. However, at our disposal there are much fewer kinds of second-order derivatives than first-order ones. This paper is concerned with first and second-order optimality conditions using approximations introduced in [1,2] as generalized derivatives. In [3,2] first-order approximations were used together with regularity conditions to establish Karush–Kuhn–Tucker type first-order conditions. Functions and mappings involved in the considered problems were assumed to be locally Lipschitz or to have upper semicontinuous bounded-valued approximations. Second-order conditions using approximations were investigated in [1] assuming that all approximations in use were compact. Possibly unbounded approximations were employed to prove optimality conditions of both orders 1 and 2 in [4–8] for various vector optimization problems, including set-valued optimization. Semicontinuity requirements were not imposed either. Instead, asymptotic pointwise compactness of approximations was assumed. This compactness is relatively relaxed, since in the finite dimensional case any approximation is asymptotically compact (in finite dimensional spaces the word "pointwise" is omitted as pointwise convergence coincides

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