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Global nonexistence results for a class of hyperbolic systems

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ABSTRACT

Our concern in this paper is to prove blow-up results to the non-autonomous nonlinear system of wave equations

$$u_{tt} - \Delta u = a(t,x)|v|^p, \qquad v_{tt} - \Delta v = b(t,x)|u|^q, \quad t > 0,\ x \in \mathbb{R}^N$$

in any space dimension. We show that a curve $\widetilde{F}(p,q) = 0$ depending on the space dimension, on the exponents p, q and on the behavior of the functions $a(t,x)$ and $b(t,x)$ exists, such that all nontrivial solutions to the above system blow-up in a finite time whenever $\widetilde{F}(p,q) > 0$. Our method of proof uses some estimates developed by Galaktionov and Pohozaev in [11] for a single non-autonomous wave equation enabling us to obtain a system of ordinary differential inequalities from which the desired result is derived. Our result generalizes some important results such as the ones in Del Santo et al. (1996) [12] and Galaktionov and Pohozaev (2003) [11]. The advantage here is that our result applies to a wide variety of problems.

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1. Introduction

The problem of the critical exponent for the nonlinear wave equation of the form

$$\begin{cases} u_{tt}(t,x) - \Delta u(t,x) = |u(t,x)|^p, & t > 0,\ x \in \mathbb{R}^N, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^N, \end{cases} \tag{1}$$

where $u_0, u_1 \in C_0^\infty$, has attracted considerable attention in the literature, see for instance [1–9] and references therein. Among the very first results establishing the critical exponent for the wave equation (1) are those of John [4]. John proved that for $N = 3$ there exists an exponent $p_0(3) = 1+\sqrt{2}$, such that if $p > p_0$, then global solutions exist for small initial data. In other words, for $p > p_0$, the effect of the nonlinearity is not so strong and problem (1) is regarded as small perturbation of the linear equation

$$u_{tt}(t,x) - \Delta u(t,x) = 0, \quad t > 0,\ x \in \mathbb{R}^N. \tag{2}$$

While if $p < p_0$, solutions blow-up in finite time. The critical exponent of (1) for N dimensions is the positive root $p_0(N)$ of the quadratic equation

$$(N-1)p^2 - (N+1)p - 2 = 0,$$

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