Nonlinear Analysis 74 (2011) 2534–2547

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Optimality conditions in nonconvex optimization via weak subdifferentials

R. Kasimbeyli [a], M. Mammadov [b,*]

[a] Department of Industrial Systems Engineering, Faculty of Engineering and Computer Sciences, Izmir University of Economics, Izmir, Turkey

[b] Centre for Informatics and Applied Optimization, University of Ballarat, Victoria, 3353, Australia

ARTICLE INFO

Article history:
Received 14 September 2010
Accepted 7 December 2010

MSC:
90C26
90C30
90C46

Keywords:
Weak subdifferential
Directional derivative
Nonconvex analysis
Optimality condition
Variational inequalities
Augmented normal cone

ABSTRACT

In this paper we study optimality conditions for optimization problems described by a special class of directionally differentiable functions. The well-known necessary and sufficient optimality condition of nonsmooth convex optimization, given in the form of variational inequality, is generalized to the nonconvex case by using the notion of weak subdifferentials. The equivalent formulation of this condition in terms of weak subdifferentials and augmented normal cones is also presented.

1. Introduction

In this paper the problem of minimizing function $f : S \to \mathbb{R}$ over the set $S \subseteq \mathbb{R}^n$ is considered.

The following is a well-known optimality condition in nonsmooth convex analysis which states that [1, Proposition 1.8.1, page 168] if $f : \mathbb{R}^n \to \mathbb{R}$ is a convex function then vector $\bar{x}$ minimizes f over a convex set $S \subset \mathbb{R}^n$ if and only if there exists a subgradient $x^* \in \partial f(\bar{x})$ such that

$$\langle x^*, x - \bar{x}\rangle \geq 0, \quad \forall\, x \in S \tag{1}$$

where

$$\partial f(\bar{x}) = \{x^* \in \mathbb{R}^n : f(x) - f(\bar{x}) \geq \langle x^*, x - \bar{x}\rangle, \text{ for all } x \in \mathbb{R}^n\} \tag{2}$$

is a subdifferential of f at $\bar{x}$. Equivalently, $\bar{x}$ minimizes f over a convex set $S \subset \mathbb{R}^n$ if and only if

$$0 \in \partial f(\bar{x}) + N_S(\bar{x}), \tag{3}$$

where

$$N_S(\bar{x}) = \{x^* \in \mathbb{R}^n : \langle x^*, x - \bar{x}\rangle \leq 0, \text{ for all } x \in S\} \tag{4}$$

* Corresponding author.
E-mail addresses: refail.kasimbeyli@ieu.edu.tr (R. Kasimbeyli), m.mammadov@ballarat.edu.au (M. Mammadov).

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.12.008