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Multiple positive solutions for semilinear elliptic equations involving multi-singular inverse square potentials and concave–convex nonlinearities

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ABSTRACT

In this paper, we deal with the existence and multiplicity of positive solutions for the multi-singular semilinear elliptic equations

$$-\Delta u-\sum_{i=1}^{k}\frac{\mu_i}{|x-a_i|^2}u=|u|^{2^*-2}u+\lambda|u|^{q-2}u,\quad x\in\Omega,$$

where $\Omega\subset\mathbb{R}^N$ $(N\geq 3)$ is a smooth bounded domain such that the points $a_i\in\Omega$, $i=1,2,\ldots,k$, $k\geq 2$, are different, $0\leq\mu_i<\bar{\mu}=(\frac{N-2}{2})^2$, $\lambda>0$, $1\leq q<2$, and $2^*=\frac{2N}{N-2}$. The results depend crucially on the parameters λ, q and μ_i for $i=1,2,\ldots,k$.

1. Introduction

In this paper, we consider the following semilinear elliptic equation:

$$\begin{cases}-\Delta u-\displaystyle\sum_{i=1}^{k}\frac{\mu_i}{|x-a_i|^2}u=|u|^{2^*-2}u+\lambda|u|^{q-2}u, & x\in\Omega,\\ u=0, & x\in\partial\Omega,\end{cases}\tag{E$_\lambda$}$$

where $\Omega\subset\mathbb{R}^N(N\geq 3)$ is a smooth bounded domain such that the points $a_i\in\Omega$, $i=1,2,\ldots,k$, $k\geq 2$, are different, $0\leq\mu_i<\bar{\mu}:=\left(\frac{N-2}{2}\right)^2$, $\lambda>0$, $1\leq q<2$, and $2^*:=\frac{2N}{N-2}$ is the critical Sobolev exponent.

In the case $\mu_i=0$, Eq. (E$_\lambda$) is related to the well known Sobolev–Hardy inequalities, essentially due to Caffarelli et al. (see [1]):

$$\left(\int_\Omega\frac{|u|^p}{|x-a|^s}\mathrm{d}x\right)^{\frac{2}{p}}\leq C_{p,s}\int_\Omega|\nabla u|^2\mathrm{d}x,\quad\forall u\in H_0^1(\Omega),$$

where $a\in\Omega$, $2\leq p\leq 2^*$ and $0\leq s<2$. For sharp constants and extremal functions, see [2]. As $p=s=2$, the above Sobolev–Hardy inequality becomes the well known Hardy inequality (see [2,3]),

$$\int_\Omega\frac{u^2}{|x-a|^2}\mathrm{d}x\leq\frac{1}{\bar{\mu}}\int_\Omega|\nabla u|^2\mathrm{d}x,\quad\forall u\in H_0^1(\Omega).\tag{1.1}$$

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