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On a class of critical singular quasilinear elliptic problem with indefinite weights

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ABSTRACT

In this paper, the eigenvalue problem for a class of quasilinear elliptic equations involving critical potential and indefinite weights is investigated. We obtain the simplicity, strict monotonicity and isolation of the first eigenvalue λ_1. Furthermore, because of the isolation of λ_1, we prove the existence of the second eigenvalue λ_2. Then, using the Trudinger–Moser inequality, we obtain the existence of a nontrivial weak solution for a class of quasilinear elliptic equations involving critical singularity and indefinite weights in the case of $0 < \lambda < \lambda_1$ by the Mountain Pass Lemma, and in the case of $\lambda_1 \leq \lambda < \lambda_2$ by the Linking Argument Theorem.

1. Introduction

In this paper, we consider the existence of a nontrivial solution of the following problem

$$\begin{cases} -\Delta_N u - \mu \dfrac{|u|^{N-2}u}{|x|^N \ln^N\left(\frac{R}{|x|}\right)} = \lambda V(x)|u|^{N-2}u + f(x,u), & x \in \Omega, \\ u = 0, \quad x \in \partial\Omega \end{cases} \tag{1.1}$$

where $\Omega \subset B_R(0)$ is a bounded domain containing 0 in $\mathbb{R}^N$, $\Delta_N u = \operatorname{div}(|\nabla u|^{N-2}u)$ denotes the N-Laplacian, $0 \leq \mu < \left(\frac{N-1}{N}\right)^N$, λ is a parameter and $V(x)$ is a given function which satisfies

$$V^+ \not\equiv 0,\ V \in L^1(\Omega), \quad V^+(x) = \max\{\pm V(x), 0\}. \tag{1.2}$$

Recently, singular quasilinear elliptic equations have been studied extensively. In 1998, Garcia Azorero and Peral [1] studied the existence of nontrivial solutions for quasilinear elliptic equations involving critical singularity

$$\begin{cases} -\Delta_p u = \lambda \dfrac{|u|^{p-2}u}{x^p} + u|u|^{r-2}u, & x \in \Omega, \\ u = 0, \quad x \in \partial\Omega. \end{cases} \tag{1.3}$$

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