



On a class of abstract neutral functional differential equations

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ABSTRACT

By using the theory of semigroups of growth α , we discuss the existence of mild solutions for a class of abstract neutral functional differential equations. A concrete application to partial neutral functional differential equations is considered.

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1. Introduction

In this paper, we study the existence of mild solutions for a class of neutral functional differential equations of the form

$$\frac{d}{dt} [x(t) + g(t, x_t)] = Ax(t) + f(t, x_t), \quad t \in [0, a], \quad (1.1)$$

$$x_0 = \varphi \in \Omega \subset \mathcal{B}, \quad (1.2)$$

where $A : D(A) \subset X \rightarrow X$ is an almost sectorial operator, $(X, \|\cdot\|)$ is a Banach space, $\mathcal{B}$ is the phase space ($\mathcal{B} = C([-r, 0], X)$ or $\mathcal{B} = L^p([-r, 0], X)$), $\Omega \subset \mathcal{B}$ is open and $g, f : [0, a] \times \Omega \rightarrow X$ are suitable functions.

There exists an extensive literature on abstract neutral differential equations treating the case in which A is a sectorial operator; see [1–6] and the references therein. Sectorial operators appear frequently in applications since many elliptic differential operators are sectorial when they are considered in Lebesgue spaces (L^p -spaces) or in spaces of continuous functions; see [7]. However, if we look at spaces of more regular functions such as the spaces of Hölder continuous functions, we find that these elliptic operators are not sectorial; see [7,8, Example 3.1.33]. Nevertheless, for these operators estimates such as

$$\|(\lambda - A)^{-1}\| \leq \frac{M}{|\lambda|^{1-\alpha}}, \quad \lambda \in \Sigma_{\omega, \theta} = \{\lambda \in \mathbb{C} : |\arg(\lambda - \omega)| < \theta\}, \quad (1.3)$$

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