



Capacity for potentials of functions in Musielak–Orlicz spaces

Fumi-Yuki Maeda^a, Yoshihiro Mizuta^b, Takao Ohno^{c,*}, Tetsu Shimomura^d

^a 4-24 Furue-higashi-machi, Nishi-ku, Hiroshima 733-0872, Japan

^b Department of Mathematics, Graduate School of Science, Hiroshima University, Higashi-Hiroshima 739-8521, Japan

^c Faculty of Education and Welfare Science, Oita University, Dannoharu Oita-city 870-1192, Japan

^d Department of Mathematics, Graduate School of Education, Hiroshima University, Higashi-Hiroshima 739-8524, Japan

ARTICLE INFO

Article history:

Received 7 June 2010

Accepted 3 June 2011

Communicated by Enzo Mitidieri

MSC:

primary 46E30

31B15

Keywords:

Capacity

Musielak–Orlicz space

Variable exponent

ABSTRACT

We define a capacity for potentials of functions in Musielak–Orlicz spaces. Basic properties of such capacity are studied. We also estimate the capacity of balls and give some applications of the estimates.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

The notion of classical Newton capacity has been generalized to various forms. Among others, Meyers [1] introduced a general notion of L^p -capacity, which is defined by general potentials of functions in the Lebesgue space L^p and such notion of capacity has been proved to provide rich results in the nonlinear potential theory as well as in the study of various function spaces and partial differential equations; see e.g., [2]. Most useful L^p -capacities are Riesz capacity and Bessel capacity, and we can estimate the capacities of balls $B(x, r)$ for these special cases, which are used to obtain relations between these capacities and Hausdorff measures (cf. the references cited above as well as [3,4]).

In [5], the notion of L^p -capacity was generalized by replacing L^p by Orlicz space. Recently, there appeared several papers dealing with capacities for special types of Orlicz spaces and estimates of the capacity of balls: [6–9].

In the mean time, variable exponent Lebesgue spaces and Sobolev spaces were introduced to discuss nonlinear partial differential equations with non-standard growth condition, and Sobolev capacity for variable exponent Sobolev space has been studied in connection with the related nonlinear potential theory: [10,11]. The Riesz capacity for the variable exponent Lebesgue space $L^{p(\cdot)}$ was considered in [12], and then that for the space $L^{p(\cdot)}(\log L)^{q(\cdot)}$ in [13].

The spaces $L^{p(\cdot)}$ and $L^{p(\cdot)}(\log L)^{q(\cdot)}$ are special cases of the Musielak–Orlicz spaces (or, generalized Orlicz spaces); see, [14,15]. The purpose of the present paper is to extend the notion of capacity to that defined by general potentials of functions in fairly general Musielak–Orlicz spaces and to show that the capacity thus defined still satisfies fundamental properties shared by those capacities stated above. We also give estimates of balls for our capacity and apply the estimates to obtain local behavior of functions in the space and to relate the capacity with a generalized harmonic measure.

* Corresponding author.

E-mail addresses: fymaeda@h6.dion.ne.jp (F.-Y. Maeda), yomizuta@hiroshima-u.ac.jp (Y. Mizuta), t-ohno@oita-u.ac.jp (T. Ohno), tshimo@hiroshima-u.ac.jp (T. Shimomura).