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Spreading speeds of a Lotka–Volterra predator–prey system: The role of the predator

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ABSTRACT

This paper is concerned with the spreading speeds of a modified Lotka–Volterra predator–prey system; the intention is to describe the propagation mode whereby the two species coinvade a new habitat. Under some assumptions, it is proved that the spreading speed of the prey is slower than in the case where the predator vanishes, and the density of the prey on the coexistence domain is also smaller compared with the case where inter-species actions disappear. Therefore, the predator has a negative effect on the evolution of the prey by slowing the spreading speed and decreasing the population density of the prey.

1. Introduction

In the mathematical models arising from nature and science, one cornerstone is the famous Lotka–Volterra predator–prey system [1,2], and one of its modified versions takes the following form:

$$\begin{cases} \dfrac{\mathrm{d}u(t)}{\mathrm{d}t} = u(t)[\lambda - au(t) - bv(t)], \\ \dfrac{\mathrm{d}v(t)}{\mathrm{d}t} = v(t)[\mu - \mathrm{d}v(t) + cu(t)], \end{cases} \tag{1.1}$$

where λ, a, b, c, d are positive constants and $\mu \in \mathbb{R}$. It is clear that u and v are in the positions of prey and predator, respectively. In particular, we assume that $\mu > 0$ holds in what follows. From the viewpoint of biology, $\mu > 0$ means that other species besides u, v may also be present in the environment where u and v are present and v has a choice of which species to feed upon (see [3]). System (1.1) is an ODE model or a mean field model, namely, the location of the individuals does not affect the evolution process of these species. However, the spatial structure of a given population or community is not homogeneously distributed due to the inhomogeneous distribution of the food that it needs and for other reasons. Therefore, different spatial effects (such as diffusion and dispersal) have been widely studied in population dynamics; we refer the reader to Malchow et al. [3] and Du and Shi [4] for some predator–prey models with spatio-temporal patterns. In particular, the corresponding reaction–diffusion model of (1.1) admits the following form:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = d_1 \triangle u(x,t) + u(x,t)[\lambda - au(x,t) - bv(x,t)], \\ \dfrac{\partial v(x,t)}{\partial t} = d_2 \triangle v(x,t) + v(x,t)[\mu - \mathrm{d}v(x,t) + cu(x,t)], \end{cases} \tag{1.2}$$

in which $x \in \Omega \subseteq \mathbb{R}^n$, $d_1, d_2 \geq 0$ and $\triangle$ is the standard Laplacian operator.

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