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Existence of three nontrivial solutions for asymptotically p-linear noncoercive p-Laplacian equations

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ABSTRACT

We consider a nonlinear elliptic problem driven by the p-Laplacian, where the right hand side nonlinearity exhibits a p-linear behavior near infinity and the Euler functional of the problem need not be coercive and, in fact, can be indefinite. Using a combination of minimax arguments with truncation techniques and Morse theory, we show that the problem has at least three nontrivial smooth solutions, two of which have a constant sign. Our method of proof uses some results on critical groups and the spectrum of the p-Laplacian, due to Perera and Dancer–Perera.

1. Introduction

Let $Z \subset \mathbb{R}^N$ be a bounded domain with a C^2-boundary ∂Z, and denote by Δ_p the p-Laplacian differential operator defined by $\Delta_p x = \operatorname{div}(\|Dx\|_{\mathbb{R}^N}^{p-2} Dx)$ for $x \in W^{1,p}(Z)$, $1 < p < \infty$. In this paper, we study the following nonlinear Dirichlet problem:

$$\begin{cases} -\Delta_p x(z) = f(z, x(z)) & \text{a.e. on } Z, \\ x|_{\partial Z} = 0. \end{cases} \tag{1}$$

The aim of this work is to establish the existence of at least three nontrivial solutions for the problem (1), when the nonlinearity $f(z,\cdot)$ exhibits an asymptotic p-linear behavior at infinity and the Euler functional of the problem need not be coercive. Recently, three solutions theorems for equations driven by the p-Laplacian, were proved by Bartsch and Liu [1], Carl and Perera [2], Degiovanni and Lancelotti [3], Liu [4], Liu and Liu [5], Papageorgiou and Papageorgiou [6], Papageorgiou et al. [7], and Zhang et al. [8]. In the works of Carl and Perera [2], Liu [4], Liu and Liu [5], Papageorgiou and Papageorgio [6], and Zhang et al. [8] (Theorem 1.1), the Euler functional is coercive. In contrast, Bartsch and Liu [1], Degiovanni and Lancelotti [3], and Papageorgiou et al. [7], they have a p-superlinear nonlinearity which satisfies the well-known Ambrosetti–Rabinowitz condition. In [8] (Theorem 1.2), it is assumed that the quotient $\frac{f(z,x)}{|x|^{p-2}x}$ has finite limits at $0^{\pm}$ and at $\pm\infty$, which are related to the Fučik spectrum of the Dirichlet p-Laplacian. The nonlinearity $x \mapsto f(x)$ is locally Lipschitz and independent of $z \in Z$.

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