Nonlinear Analysis 74 (2011) 2563–2575

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Oscillation criteria for half-linear elliptic inequalities with $p(x)$-Laplacians via Riccati method☆

Norio Yoshida*

Department of Mathematics, University of Toyama, Toyama 930-8555, Japan

ARTICLE INFO

Article history:
Received 8 October 2010
Accepted 8 December 2010

MSC:
35B05
35J92

Keywords:
Oscillation
$p(x)$-Laplacian
Riccati inequality
Half-linear
Elliptic inequalities

ABSTRACT

Riccati inequality for half-linear elliptic inequalities with $p(x)$-Laplacians is established, and oscillation criteria are derived by using the Riccati inequality. Our method is to reduce oscillation problems for half-linear elliptic inequalities with $p(x)$-Laplacians to one-dimensional Riccati inequality with variable exponents. Generalizations to more general elliptic inequalities with $p(x)$-Laplacians are also discussed.

1. Introduction

Beginning with the work of Noussair and Swanson [1] dealing with semilinear elliptic inequalities, efforts have been made by numerous authors to generalize to more general elliptic equations, for example, quasilinear elliptic equations including half-linear elliptic equations.

In recent years there has been considerable investigation on oscillations of half-linear elliptic equations. In 1998 Usami [2] investigated oscillations of half-linear elliptic equations of the form

$$\nabla \cdot \left(a(x)|\nabla u|^{p-2}\nabla u\right) + c(x)|u|^{p-2}u = 0 \quad (p > 1)$$

by using Riccati techniques, where $\nabla = (\partial/\partial x_1, \ldots, \partial/\partial x_n)$ and the dot $\cdot$ denotes the scalar product, and then oscillation results via Riccati method have been developed by several authors, see for example, [3–9], and the references cited therein.

The operator $-\nabla \cdot \left(|\nabla u|^{p(x)-2}\nabla u\right)$ is said to be $p(x)$-Laplacian, and becomes p-Laplacian $-\nabla \cdot \left(|\nabla u|^{p-2}\nabla u\right)$ if $p(x) = p$ (constant). Much current interest has been focused on various mathematical problems with variable exponent growth conditions (see [10]). The study of such problems arises from nonlinear elasticity theory, electrorheological fluids (cf. [11,12]). We mention, in particular, the paper [13] by Allegretto in which Picone identity arguments are used, and the formulae that are closely related to Picone identities and Riccati inequalities are established.

Existence of weak solutions of $p(x)$-Laplacian equations of the form

$$-\nabla \cdot \left(a(x)|\nabla u|^{p(x)-2}\nabla u\right) + |u|^{p(x)-2}u = f(x,u) \quad \text{in } \mathbb{R}^n$$

were investigated by several authors, see for example, [14–17].

☆ This research was partially supported by Grant-in-Aid for Scientific Research (C) (No. 20540159), the Ministry of Education, Culture, Sports, Science and Technology, Japan.

* Tel.: +81 76 445 6562; fax: +81 76 445 6573.
E-mail address: nori@sci.u-toyama.ac.jp.

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.12.011