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Zero relaxation limit to centered rarefaction waves for Jin–Xin relaxation system

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ABSTRACT

In this paper, we study the zero relaxation limit problem for the following Jin–Xin relaxation system

$$\begin{cases} u_t + v_x = 0 \\ v_t + a^2 u_x = \dfrac{1}{\epsilon}(f(u) - v) \end{cases} \tag{E}$$

with initial data

$$(u, v)(x, 0) = (u_0(x), v_0(x)) \to (u_\pm, v_\pm), \quad u_\pm > 0, \text{ as } x \to \pm\infty. \tag{I}$$

This system was proposed by Jin and Xin (1995) [1] with an interesting numerical origin. As the relaxation time tends to zero, this system converges to the equilibrium conservation law formally. Our interest is to study the case where the initial data are allowed to have jump discontinuities such that the corresponding solutions to the equilibrium conservation law contain centered rarefaction waves and the limits $(u_\pm, v_\pm)$ of the initial data at $x = \pm\infty$ do not satisfy the equilibrium equation, i.e., $v_\pm \neq f(u_\pm)$. In particular, Riemann data connected by rarefaction curves are included. We show that if the wave strength is sufficiently small, then the solution for the relaxation system exists globally in time and converges to the solution of the corresponding rarefaction waves uniformly as the relaxation time goes to zero except for an initial layer.

1. Introduction

In this paper, we investigate the asymptotic behaviour of the following Jin–Xin relaxation system

$$\begin{cases} u_t + v_x = 0 \\ v_t + a^2 u_x = \dfrac{1}{\epsilon}(f(u) - v) \end{cases} \tag{1.1}$$

as the relaxation time ϵ goes to zero. Here u and v are unknown functions of $t > 0$ and $x \in R$, and $f(u)$ is a smooth function of u. We assume that the initial function satisfies

$$(u, v)(x, 0) = (u_0(x), v_0(x)) \to (u_\pm, v_\pm), u_\pm > 0, \quad \text{as } x \to \pm\infty. \tag{1.2}$$

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