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On resonant Neumann problems: Multiplicity of solutions

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ABSTRACT

We consider a semilinear Neumann problem with an asymptotically linear reaction term. We assume that resonance occurs at infinity. Using variational methods based on the critical point theory, together with the reduction technique and Morse theory, we show that the problem has at least four nontrivial smooth solutions.

1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$ be a bounded domain with a C^2 boundary $\partial\Omega$. In this paper, we study the existence of multiple nontrivial smooth solutions when resonance occurs at infinity, for the following semilinear Neumann problem:

$$\begin{cases} -\triangle u(z) = f(z, u(z)) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

Here $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is measurable, C^1 in the $x \in \mathbb{R}$ variable, asymptotically linear at infinity and resonance occurs with respect to a nonprincipal eigenvalue λ_m, $m \geq 1$, of $(-\triangle, H^1(\Omega))$ (in what follows, by $\{\lambda_k\}_{k\geq 0}$ we denote the distinct eigenvalues of $(-\triangle, H^1(\Omega))$). Asymptotically linear problems were studied primarily within the framework of Dirichlet equations, starting with the pioneering work of Amann and Zehnder [1], who proved that if $f(z, x) = f(x)$ is a C^1-function, $\lim\limits_{|x|\to\infty} \frac{f(x)}{x} = \lambda \notin \sigma_D(2)$ ($\sigma_D(2)$ denoting the Dirichlet spectrum of $(-\triangle, H_0^1(\Omega))$) and there exists at least one element of $\sigma_D(2)$ between λ and $\lambda + f'(0)$, then the Dirichlet problem has at least one nontrivial solution. Similar results, using alternative approaches, were also obtained by Chang [2] and Lazer and Solimini [3]. Since then, there have been several more papers dealing with asymptotically linear Dirichlet problems. We mention the works of Hirano and Nishimura [4], Landesman et al. [5], Liu [6], Li and Zhou [7], Li and Perera [8], Robinson [9] and the references therein.

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