Nonlinear Analysis 74 (2011) 2439–2447

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Remarks on some basic concepts in the KKM theory

Sehie Park *

The National Academy of Sciences, Republic of Korea, Seoul 137–044, Republic of Korea
Department of Mathematical Sciences, Seoul National University, Seoul 151–747, Republic of Korea

ARTICLE INFO

Article history:
Received 28 January 2010
Accepted 18 October 2010

MSC:
47H04
47H10
49J40
52A99
54C65
54H25
58E35
90D13
91B50

Keywords:
KKM type theorems
Fixed point
Abstract convex space
Generalized convex (G-convex) space
H-space
Partial KKM principle
KKM space

ABSTRACT

In the KKM theory, some authors adopt the concepts of the compact closure (ccl), compact interior (cint), transfer compactly closed-valued multimap, transfer compactly l.s.c. multimap, and transfer compactly local intersection property, respectively, instead of the closure, interior, closed-valued multimap, l.s.c. multimap, and possession of a finite open cover property. In this paper, we show that such adoption is inappropriate and artificial. In fact, any theorem with a term with "transfer" attached is equivalent to the corresponding one without "transfer". Moreover, we can invalidate terms with "compactly" attached by giving a finer topology on the underlying space. In such ways, we obtain simpler formulations of KKM type theorems, Fan–Browder type fixed point theorems, and other results in the KKM theory on abstract convex spaces.

1. Introduction

The KKM theory, first called this by the author [1], is the study of applications of various equivalent or generalized formulations of the KKM theorem given by Knaster, Kuratowski, and Mazurkiewicz in 1929. At the beginning, the theory was mainly devoted to the study of convex subsets of topological vector spaces (t.v.s.). Later, it was extended to convex spaces by Lassonde, and to C-spaces (or H-spaces) by Horvath, and others. In 1993–2006, the KKM theory was further extended to generalized convex (G-convex) spaces in a sequence of papers by the present author and others; see [2,3] and references therein. Furthermore, since 2006, we have introduced a new concept of abstract convex spaces which are adequate for establishing the KKM theory; see [3–7].

The "closed" version of the following is the origin of the KKM theory; see [2].

Theorem 1.1 (*KKM*)**.** *Let D be the set of vertices of an n-simplex Δ_n and $G : D \multimap \Delta_n$ be a KKM map (that is,* $\mathrm{co}\, A \subset G(A)$ *for each $A \subset D$) with closed (or open) values. Then $\bigcap_{z\in D} G(z) \neq \emptyset$.*

* Corresponding address: The National Academy of Sciences, Republic of Korea, Seoul 137–044, Republic of Korea. Fax: +82 2 887 4694.
E-mail addresses: shpark@math.snu.ac.kr, parkcha38@hanmail.net.

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.10.035