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High order stability conditions for degenerate optimization problems; elements of p-regularity theory

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ABSTRACT

This paper is devoted to sensitivity analysis of degenerate equality-constrained optimization problems, when the customary regularity of constraints may be violated. Under the p-regularity assumptions we develop p-order stability theory for this class of problems subject to small perturbations and present the estimations of asymptotic behavior of the solutions to the perturbed problems. We derive conditions that guarantee the existence of a solution for perturbed optimization problems and substantiate the assertion of natural character of these conditions. All results are illustrated by classical calculus of variations problems.

1. Introduction

In this paper we consider the following optimization problem:

$$\min \varphi(x) \quad \text{subject to } F(x) = 0, \tag{1}$$

where we have $F : X \to Y$, with X and Y Banach spaces, $\varphi : X \to \mathbb{R}^1$, $F \in \mathcal{C}^{p+2}(X)$, $p \geq 1$, $p \in \mathbb{N}$, $\varphi \in \mathcal{C}^2(X)$ and moreover at the solution point x^* of problem (1) the following condition:

$$\mathrm{Im} F'(x^*) \neq Y \tag{2}$$

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