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Multiplicity of positive solutions for a semilinear p-Laplacian system with Sobolev critical exponent

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ABSTRACT

In this paper, we study the semilinear p-Laplacian system with critical growth terms in bounded domains. By using the Nehari manifold and variational methods, we prove that the system has at least two positive solutions when the pair of the parameters (λ, μ) belongs to a certain subset of R^2.

1. Introduction

In this paper, we consider the multiplicity results of positive solutions of the following semilinear p-Laplacian system:

$$\begin{cases} -\Delta_p u = \dfrac{1}{p^*}\dfrac{\partial F(x,u,v)}{\partial u} + \lambda|u|^{q-2}u & \text{in } \Omega, \\ -\Delta_p v = \dfrac{1}{p^*}\dfrac{\partial F(x,u,v)}{\partial v} + \mu|v|^{q-2}v & \text{in } \Omega, \\ u > 0, v > 0 & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $0 \in \Omega$ is a bounded domain in R^N with smooth boundary $\partial\Omega$, $F \in C^1(\overline{\Omega} \times (R^+)^2, R^+)$ is positively homogeneous of degree p^*($p^* = \frac{pN}{N-p}$ denotes the Sobolev critical exponent.), that is, $F(x, tu, tv) = t^{p^*}F(x, u, v)(t > 0)$ holds for all $(x, u, v) \in \overline{\Omega} \times (R^+)^2$, $\left(\frac{\partial F(x,u,v)}{\partial u}, \frac{\partial F(x,u,v)}{\partial v}\right) = \nabla F$. We assume that $1 < q < p < N, \lambda > 0,\ \mu > 0$.

In recent years, there have been many papers concerned with the existence and multiplicity of positive solutions for semilinear elliptic problems. Results relating to these problems can be found in Wu [1–3], Garcia-Azorero et al. [4], Furtado and Paiva [5], and the references therein.

Brown and Wu [6] considered the following semilinear elliptic system:

$$\begin{cases} -\Delta u + u = \dfrac{\alpha}{\alpha+\beta} f(x)|u|^{\alpha-2}u|v|^{\beta} & \text{in } \Omega, \\ -\Delta v + v = \dfrac{\beta}{\alpha+\beta} f(x)|u|^{\alpha}|v|^{\beta-2}v & \text{in } \Omega, \\ \dfrac{\partial u}{\partial n} = \lambda g(x)|u|^{q-2}u, \quad \dfrac{\partial v}{\partial n} = \mu h(x)|v|^{q-2}v & \text{on } \partial\Omega, \end{cases} \tag{1.2}$$

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