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Gauss curvature estimates for the convex level sets of a minimal surface immersed in $\mathbb{R}^{n+1}$ ☆

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ABSTRACT

This paper is concerned with the Gauss curvature estimates for convex level sets of a minimal surface immersed in $\mathbb{R}^{n+1}$. It is proved that a function involving the Gauss curvature of a level set attains its minimum on the boundary of the minimal surface. As an application, for a minimal graph on a convex ring, a positive lower bound for the Gauss curvature of the convex level set in terms of the Gauss curvature of the boundary and the norm of graph gradient on the boundary can be given.

1. Introduction

The convexity of level sets of the solutions to elliptic partial differential equations is a classical subject. For instance, Ahlfors [1] contains the well-known result that level curves of a Green function on a simply connected convex domain in the plane are convex Jordan curves. In 1931, Gergen [2] proved the star-shapedness of the level sets of a Green function on a three-dimensional star-shaped domain. In 1956, Shiffman [3] studied the minimal annulus in $\mathbb{R}^3$ whose boundary consists of two closed convex curves in parallel planes P_1, P_2. He proved that the intersection of the surface with any parallel plane P between P_1 and P_2 is a convex Jordan curve. In 1957, Gabriel [4] proved that the level sets of a Green function on a three-dimensional bounded convex domain are strictly convex; see also the book by Hörmander [5]. Lewis [6] extended Gabriel's result for p-harmonic functions to higher dimensions. Caffarelli and Spruck [7] generalized the results to a class of semilinear elliptic partial differential equations. Using the idea of Caffarelli and Friedman [8], Korevaar [9] gave a new proof of the results of [6,7] by applying the constant rank theorem of the second fundamental form for the convex level sets of a p-harmonic function. A survey of this subject is given by Kawohl [10]. For more recent generalizations, the readers can see the papers by Bianchini et al. [11] and Bian et al. [12].

Now we turn to the curvature estimates for the level sets of elliptic partial differential equations. For a two-dimensional harmonic function and a minimal surface with convex level curves, Ortel and Schneider [13] and Longinetti [14,15] proved that the curvature of the level curves attains its minimum on the boundary. There are some recent generalizations to higher dimensions; we list some of them in the following. Jost et al. [16] and Ma et al. [17] proved that the Gauss curvature and the

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