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An optimal Liouville-type theorem of the quasilinear parabolic equation with a p-Laplace operator

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ABSTRACT

In this paper, we consider nonnegative solutions of the quasilinear parabolic equation with p-Laplace operator $u_t = \mathrm{div}(|\nabla u|^{p-2}\nabla u) + |u|^{q-1}u$, where $p > 2$ and $q > p-1$. Our main result is that there is no nontrivial positive bounded radial entire solution. The proof is based on intersection comparison arguments, which can be viewed as a sophisticated form of the maximum principle and has been used to deal with the semilinear heat equation by Poláčik and Quittner [Peter Poláčik, Pavol Quittner, A Liouville-type theorem and the decay of radial solutions of a semilinear heat equation, Nonlinear Analysis TMA 64 (2006) 1679–1689] and the porous medium equation by Souplet [Ph. Souplet, An optimal Liouville-type theorem for radial entire solutions of the porous medium equation with source, J. Differential Equations 246 (2009) 3980–4005].

1. Introduction

In this paper, we study nonnegative solutions of the p-Laplace heat equation with a source term:

$$u_t = \mathrm{div}(|\nabla u|^{p-2}\nabla u) + |u|^{q-1}u, \quad x \in \mathbb{R}^N,\ t \in \mathbb{R}, \tag{1.1}$$

where $2 < p < N$ and $p-1 < q < Np/(N-p)-1$.

We are particularly interested in the classical radial solutions $u(r,t) = u(|x|,t)$ contained in $C^{2,1}((0,\infty)\times\mathbb{R}) \cap C^{1,1/2}([0,\infty)\times\mathbb{R})$; below, we refer to such solutions as entire solutions. Then we address the question of the existence of positive bounded radial entire solutions. Indeed, it is well known (cf. [1] and see, e.g., [2]) that for $q \geq Np/(N-p)-1$, the equation

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = u^q, \quad x \in \mathbb{R}^N. \tag{1.2}$$

admits positive classical solutions that are bounded and radially symmetrical. On the contrary, (1.1) has no positive stationary solution, when $p-1 < q < Np/(N-p)-1$, as a consequence of the nonexistence of nontrivial solutions $u \geq 0$ of (1.2) in this case (see [1]). The latter nonexistence statement is usually referred to as an elliptic Liouville-type theorem. For Liouville-type results for parabolic equations, particularly, the equations $u_t = \Delta u + u^p$ and $u_t = \Delta(u^m) + u^p$, we refer the readers to [3,4]. The more general question of parabolic Liouville-type properties, i.e., the nonexistence of non-stationary entire solutions to (1.1) has not yet been considered in the whole subcritical range $p-1 < q < Np/(N-p)-1$. In this paper, we mainly prove the nonexistence of positive bounded radial solutions for Eq. (1.1).

Parabolic equations of p-Laplace type arise in many applications in the fields of mechanics, physics and biology (non-Newtonian fluids, gas flows in porous media, spread of biological populations, etc.) (cf.[5]). The mathematical model (1.1) is

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