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Existence of solitary wave solutions for the nonlinear Klein–Gordon equation coupled with Born–Infeld theory with critical Sobolev exponent

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ABSTRACT

In this paper, we prove the existence of solutions for the nonlinear Klein–Gordon equation coupled with Born–Infeld theory under the electrostatic solitary wave ansatz by using variational methods.

1. Introduction

In this paper, we are concerned with the existence of solitary wave solutions of the following Klein–Gordon equation coupled with Born–Infeld theory

$$\begin{cases} \Delta u = (m^2 - (\omega + \phi)^2)u - |u|^{q-2}u - |u|^{2^*-2}u & \text{in } \mathbb{R}^3, \\ \Delta\phi + \beta\Delta_4\phi = 4\pi(\omega + \phi)u^2 & \text{in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

and

$$\begin{cases} \nabla \cdot \dfrac{\nabla\phi}{\sqrt{1 - \frac{1}{b^2}|\nabla\phi|^2}} = u^2(\omega + \phi) & \text{in } \mathbb{R}^3, \\ \Delta u = (m^2 - (\omega + \phi)^2)u - |u|^{q-2}u - |u|^{2^*-2}u & \text{in } \mathbb{R}^3. \end{cases} \tag{1.2}$$

Such classes of equations can be deduced by coupling the Klein–Gordon equation

$$\psi_{tt} - \Delta\psi + m^2\psi - |\psi|^{q-2}\psi - |\psi|^{2^*-2}\psi = 0, \tag{1.3}$$

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