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Brake type closed characteristics on reversible compact convex hypersurfaces in $\mathbf{R}^{2n}$

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In this paper, let Σ be a C^3 compact convex in $\mathbf{R}^{2n}$ satisfying the reversible condition $N\Sigma = \Sigma$ with $N = \mathrm{diag}(-I_n, I_n)$. We prove that if there are exactly n geometrically distinct closed characteristics on Σ and all of them are nondegenerate, then all of them must be brake orbits up to a suitable translation of time. Moreover, for $n = 2$ or 3, we prove that if there are exactly n geometrically distinct closed characteristics on Σ, then all of them must be brake orbits up to a suitable translation of time.

1. Introduction and main results

In this paper, let $J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$ and $N = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$ with I being the identity in $\mathbf{R}^n$. Let Σ be a C^3 compact hypersurface in $\mathbf{R}^{2n}$ bounding a bounded and strictly convex domain C in $\mathbf{R}^{2n}$. Without loss of generality, we may assume that C contains the origin. We denote the set of all such hypersurfaces in $\mathbf{R}^{2n}$ by $\mathcal{H}(2n)$. We call Σ reversible if it satisfies the reversible condition $\Sigma = N\Sigma := \{Nx | x \in \Sigma\}$ and denote by $\mathcal{H}_b(2n)$ the set of all reversible hypersurfaces in $\mathcal{H}(2n)$. We consider the dynamics problem of finding $\tau > 0$ and an absolutely continuous curve $x : [0, \tau] \to \mathbf{R}^{2n}$ such that

$$\dot{x}(t) = JN_\Sigma(x(t)), \quad x(t) \in \Sigma, \tag{1.1}$$

$$x(\tau + t) = x(t), \quad \forall t \in \mathbf{R}, \tag{1.2}$$

where $N_\Sigma(x)$ is the outward unit vector at x of Σ.

A solution (τ, x) of problem (1.1)–(1.2) is called a closed characteristic on Σ. Furthermore, as in [1], if the closed characteristic (τ, x) satisfies that $x(-t) = Nx(t)$ for all $t \in \mathbf{R}$, we call it a brake orbit on Σ.

Two closed characteristics (τ_1, x_1) and (τ_2, x_2) are called geometrically distinct if $x_1(\mathbf{R}) \neq x_2(\mathbf{R})$. We denote by $\mathcal{J}(\Sigma)$ and $\tilde{\mathcal{J}}(\Sigma)$ the set of all closed characteristics (τ, x) on Σ with τ being the minimal period of x and geometrically distinct ones on Σ respectively. For $(\tau, x) \in \mathcal{J}(\Sigma)$, we denote by $[(\tau, x)]$ the set of all elements in $\mathcal{J}(\Sigma)$ which are geometrically the same as (τ, x).

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