Nonlinear Analysis 74 (2011) 1200–1211

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Gradient estimates for parabolic $\mathcal{A}$-harmonic equations

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ARTICLE INFO

Article history:
Received 14 September 2010
Accepted 29 September 2010

MSC:
35K55
35K65

Keywords:
Gradient estimates
Parabolic
$\mathcal{A}$-harmonic
Orlicz spaces
Sobolev spaces
Nonlinear

ABSTRACT

In this paper we obtain gradient estimates in Sobolev spaces and Orlicz spaces for weak solutions of parabolic $\mathcal{A}$-harmonic equations under the assumptions that $\mathcal{A}$ satisfies some proper conditions and the given function satisfies some moderate growth condition.

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1. Introduction

Assume that Ω is an open bounded domain in $\mathbb{R}^n$ for $n > 2$ and $p > \frac{2n}{n+2}$. We then consider the following general nonlinear parabolic problem

$$u_t - \operatorname{div} \mathcal{A}(\nabla u, x, t) = \operatorname{div}(|\mathbf{f}|^{p-2}\mathbf{f}) \quad \text{in } \Omega_T =: \Omega \times (0, T], \tag{1.1}$$

where $\mathbf{f} = (f^1, \ldots, f^n)$ and $\mathcal{A} = \mathcal{A}(\xi, x, t) : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ are two given vector fields, and $\mathcal{A}$ is measurable in (x, t) for each ξ and continuous in ξ for almost everywhere (x, t). Moreover, for given $p \in (1, \infty)$ the structural conditions on the function $\mathcal{A}(\xi, x, t)$ are given as follows:

$$[\mathcal{A}(\xi, x, t) - \mathcal{A}(\eta, x, t)] \cdot (\xi - \eta) \geq \begin{cases} C_1|\xi - \eta|^p, & p \geq 2; \\ C_1(|\xi| + |\eta|)^{p-2}|\xi - \eta|^2, & 1 < p < 2, \end{cases} \tag{1.2}$$

$$|\mathcal{A}(\xi, x, t)| \leq C_2(1 + |\xi|^{p-1}), \tag{1.3}$$

$$\mathcal{A}(\xi, x, t) \cdot \xi \geq C_3|\xi|^p - C_4, \tag{1.4}$$

$$|\mathcal{A}(\xi, x, t) - \mathcal{A}(\xi, y, s)| \leq C_5 w(|x - y| + |t - s|^{1/2})(1 + |\xi|^{p-1}) \tag{1.5}$$

for all $\xi, \eta \in \mathbb{R}^n$, $(x, t), (y, s) \in \Omega_T$ and some positive constants $C_i > 0$, $i = 1, 2, 3, 4, 5$. Here the modulus of continuity $w(x) : \mathbb{R}^+ \to \mathbb{R}^+$ is non-decreasing and satisfies

$$w(r) \to 0 \quad \text{as } r \to 0. \tag{1.6}$$

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doi:10.1016/j.na.2010.09.060