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Positive solutions for a fourth order p-Laplacian boundary value problem☆

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ABSTRACT

In this paper, we study the existence, multiplicity and uniqueness of positive solutions for the fourth order p-Laplacian boundary value problem

$$\begin{cases}(|u''|^{p-1}u'')'' = f(t,u),\\ u^{(2i)}(0) = u^{(2i)}(1) = 0, \quad i = 0, 1.\end{cases}$$

Here $p > 0$ and $f \in C([0,1]\times\mathbb{R}^+,\mathbb{R}^+)$ ($\mathbb{R}^+ := [0,\infty)$). Based on a priori estimates achieved by utilizing properties of concave functions, we use fixed point index theory to establish our main results.

1. Introduction

In this paper, we study the existence, multiplicity and uniqueness of positive solutions for the fourth order p-Laplacian boundary value problem

$$\begin{cases}(|u''|^{p-1}u'')'' = f(t,u), \quad t\in(0,1),\\ u^{(2i)}(0) = u^{(2i)}(1) = 0, \quad i = 0, 1,\end{cases} \tag{1.1}$$

where $p > 0$ and $f \in C([0,1]\times\mathbb{R}^+,\mathbb{R}^+)$ ($\mathbb{R}^+ := [0,\infty)$). Note that by a positive solution of (1.1) we mean a function $u \in C^2[0,1]\cap C^4(0,1)$ that solves (1.1) and satisfies $|u''|^{p-1}u'' \in C^2(0,1)$ and $u(t) > 0$, $t \in (0,1)$.

Second order differential equations with the p-Laplacian operator arise in modeling different physical and natural phenomena, which can be encountered in, for instance, non-Newtonian mechanics, nonlinear elasticity, glaciology, population biology, combustion theory, and nonlinear flow laws, see [1,2]. This explains why many papers have been published on existence of solutions for differential equations with the p-Laplacian operator; see, for instance, [3–14] and references therein. Fourth order boundary value problems, including those with the p-Laplacian operator, have their origin in beam theory [15,16], ice formation [17,18], fluids on lungs [19], brain warping [20,21], designing special curves on surfaces [22,20], etc. In beam theory, more specifically, a beam with a small deformation, a beam of a material which

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