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Energy decay rates via convexity for some second-order evolution equation with memory and nonlinear time-dependent dissipation

Shun-Tang Wu *

General Education Center, National Taipei University of Technology, Taipei, 106, Taiwan

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ABSTRACT

The stabilization of the following abstract integro-differential equation:

$$u''(t) + Au(t) + \int_0^t g(t-s)Au(s)\mathrm{d}s + Q(t, u'(t)) = \nabla F(u(t)),$$

is investigated. We establish the general decay rate of the solution energy in terms of the behavior of the nonlinear feedback and the relaxation function g, without imposing any restrictive growth assumption on the damping at the origin and strongly weakening the usual assumption of the relaxation function g. Our approach is based on the multiplier method and make use of some properties of the convex functions. These decay results can be applied to various concrete models. We shall study some examples to illustrate our result.

1. Introduction

We know that viscoelastic materials present memory effects. These properties are due to the mechanical response influenced by their past history of the materials themselves. From the mathematical point of view, these memory effects are modeled by integro-differential equations. Questions related to the longtime behavior of the solutions for the PDE system have attracted considerable attention in recent years. A natural question concerning the longtime behavior is about the decay rate of the solutions to those models. In this paper, we will study the asymptotic behavior of solutions for the integro-differential equation and we shall establish an explicit decay rate of the solution energy.

We consider the following abstract integro-differential equation:

$$\begin{aligned} &u''(t) + Au(t) + \int_0^t g(t-s)Au(s)\mathrm{d}s + Q(t, u'(t)) = \nabla F(u(t)), \quad t > 0, \\ &u(0) = u_0, \quad u'(0) = u_1, \end{aligned} \tag{1.1}$$

in a Hilbert space $X = L^2(\Omega)$, where Ω is a bounded domain in R^n with smooth boundary $\partial\Omega$. Here, $A : D(A) \subset X \to X$ is a corceive self-adjoint linear operator with dense domain $D(A)$, ∇F denotes the gradient of a Gâteaux differentiable functional $F : D(\sqrt{A}) \to R$, $Q : (0, \infty) \times X \to X$ is a nonlinear operator which is understood to be the damping term, and g represents the kernel of the memory term, with conditions to be stated later. The class introduced in (1.1) subsumes a variety of models

* Tel.: +886 2 22401878.
E-mail address: stwu@ntut.edu.tw.

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