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Utility-based indifference pricing in regime-switching models

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ABSTRACT

In this paper, we study utility-based indifference pricing and hedging of a contingent claim in a continuous-time, Markov, regime-switching model. The market in this model is incomplete, so there is more than one price kernel. We specify the parametric form of price kernels so that both market risk and economic risk are taken into account. The pricing and hedging problem is formulated as a stochastic optimal control problem and is discussed using the dynamic programming approach. A verification theorem for the Hamilton–Jacobi–Bellman (HJB) solution to the problem is given. An issuer's price kernel is obtained from a solution of a system of linear programming problems and an optimal hedged portfolio is determined.

1. Introduction

The work of Black and Scholes [1] and Merton [2] provided a solution in simple situations to derivative valuation and hedging. They considered the situation in which the price dynamics of the underlying risky asset are described by a geometric Brownian motion (GBM). Together with the assumptions of a perfect market and the absence of arbitrage, they were able to perfectly replicate the payoff of a derivative security by continuously rebalancing a portfolio consisting of a bond and a share, and so derived an explicit pricing formula for a standard European call option. The advantage of the Black–Scholes–Merton formula is that it is preference free; that is, it does not depend on a choice of utility function and the appreciation rate of the underlying risky asset. Therefore, the subjective views of market participants about the appreciation rate and risk preferences do not influence the option price in the Black–Scholes–Merton world. Despite its compact form and popularity, the assumptions underlying the Black–Scholes–Merton model are often questioned. The GBM cannot explain some observed important empirical features of asset price dynamics. In the past three decades, a number of option pricing models based on more realistic price dynamics have been proposed. These include the jump–diffusion model, the stochastic volatility model, the GARCH option pricing model, amongst others. These models can provide a more realistic way to describe key empirical features of share price data, such as heavy-tailedness, and the option price data, such as the implied volatility smile or smirk. However, they do not take into account structural changes in economic conditions when modeling asset price dynamics and valuing options. In practice, structural changes in economic conditions have a significant impact on and

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