



Multiple solutions for a class of biharmonic elliptic systems with Sobolev critical exponent

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ABSTRACT

In this paper, we study the existence and multiplicity of nontrivial solutions for a class of biharmonic elliptic systems with Sobolev critical exponent in a bounded domain. By using the variational method and the Nehari manifold, we obtain the existence and multiplicity results of nontrivial solutions for the systems.

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1. Introduction and main results

In this paper, we consider the existence and multiplicity of nontrivial solutions for the following critical biharmonic elliptic systems:

$$\begin{cases} \Delta^2 u = F_u(u, v) + \lambda |u|^{q-2} u, & x \in \Omega, \\ \Delta^2 v = F_v(u, v) + \delta |v|^{q-2} v, & x \in \Omega, \\ u = \frac{\partial u}{\partial n} = 0, & v = \frac{\partial v}{\partial n} = 0, & x \in \partial\Omega, \end{cases} \quad (1.1)$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 5$) is a bounded domain with smooth boundary $\partial\Omega$, $0 \in \Omega$, $F \in C^1(\mathbb{R}^2, \mathbb{R}^+)$ is homogeneous of degree 2^* ($2^* = \frac{2N}{N-4}$ denotes the Sobolev critical exponent), that is, $F(tu, tv) = t^{2^*} F(u, v)$ holds for all $(u, v) \in \mathbb{R}^2$, $(F_u(u, v), F_v(u, v)) = \nabla F$, $\lambda > 0$, $\delta > 0$, $1 < q < 2$, $\frac{\partial}{\partial n}$ is the outer normal derivative.

In recent years, there have been many papers concerned with the existence and multiplicity of nontrivial solutions for nonlinear elliptic systems. The results relating to these problems can be found in [1–7], and the references therein. In [8], Wu considered the following semilinear elliptic system with subcritical nonlinearity of concave–convex type and sign-changing

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