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Global existence and blowup of a localized problem with free boundary☆

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ABSTRACT

This paper is concerned with a double fronts free boundary problem for the heat equation with a localized nonlinear reaction term. The local existence and uniqueness of the solution are given by applying the contraction mapping theorem. Then we present some conditions so that the solution blows up in finite time. Finally, the long-time behavior of the global solution is discussed. We show that the solution is global and fast if the initial data is small and that a global slow solution is possible when the initial data is suitably large.

1. Introduction

In this paper, we investigate the behavior of the positive solution $u(t,x)$ of the following localized problem with double fronts free boundary

$$\begin{cases} u_t - \mathrm{d}u_{xx} = u^p(t,0), & t>0,\ g(t)<x<h(t), \\ u(t,g(t)) = 0, & g'(t) = -\mu u_x(t,g(t)), \quad t>0, \\ u(t,h(t)) = 0, & h'(t) = -\mu u_x(t,h(t)), \quad t>0, \\ g(0) = -h_0, & h(0) = h_0, \quad u(0,x) = u_0(x), \quad -h_0 \le x \le h_0, \end{cases} \tag{1.1}$$

where both $x=g(t)$ and $x=h(t)$ are moving boundaries to be determined, $h_0>0$, $P>1$, d and μ are positive constants, and the initial function u_0 satisfies

$$\begin{cases} u_0 \in C^2([-h_0,h_0]), \\ u_0(-h_0) = u_0(h_0) = 0, \quad \text{and } u_0>0 \text{ in } (-h_0,h_0). \end{cases} \tag{1.2}$$

In [1,2], the authors considered a Stefan problem with nonlocal superlinear term, and exhibited an energy condition under which the solution would blow up in finite time. In addition, they proved that all global solutions are bounded and decay uniformly to 0. Moreover, they showed that the free boundary converges to a finite limit and the solution decays at an exponential rate, or the free boundary grows up to infinity and the decay rate is at most polynomial.

For the localized equation

$$u_t - \Delta u = f(u(t,x_0)) \tag{1.3}$$

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