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# Multiple solutions for a semilinear problem with combined terms and nonlinear boundary conditions

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ABSTRACT

We consider the problem

$$-\Delta u + u = f(x,u) \quad \text{in } \Omega, \qquad \frac{\partial u}{\partial \eta} = h(x)|u|^{q-2}u \quad \text{on } \partial\Omega,$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 3$, $1 \leq q < 2$ and $h$ belongs to an appropriated Lebesgue space. In our main results, we suppose that $f$ is an asymptotically linear function and we obtain multiplicity of solutions when the norm of $h$ is small. We also present a multiplicity result in the case that $f$ is nonquadratic at infinity.



## 1. Introduction

In this paper, we consider the problem

$$\begin{cases} -\Delta u + u = f(x,u) & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \eta} = h(x)|u|^{q-2}u & \text{on } \partial\Omega, \end{cases} \tag{$P$}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 3$ and $\frac{\partial}{\partial \eta}$ is the outer normal derivative. The function $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function with subcritical growth. More specifically, if we denote by $\sigma'$ the Hölder conjugate of $\sigma > 1$, we assume that $f$ satisfies the following condition

$(f_0)$ there exist $2 < p < 2^*$, $a_1 > 0$ and $a \in L^{\sigma_p}(\Omega)$ such that

$$|f(x,s)| \leq a_1|s|^{p-1} + a(x), \quad \text{for a.e. } x \in \Omega,\ s \in \mathbb{R},$$

where $2^* := 2N/(N-2)$ and $\sigma_p := (2^*/p)'$;

concerning the term on the boundary, we assume that $1 \leq q < 2$ and

$(h_0)$ $h \in L^{\sigma_q}(\partial\Omega)$, where $2_* := 2(N-1)/(N-2)$ and $\sigma_q := (2_*/q)'$.

We say that $f$ is asymptotically linear if there exists a function $k$ such that

$$\lim_{|s|\to\infty} \frac{f(x,s)}{s} = k(x).$$

In the Dirichlet case, it is well known (see [1–4]) that the existence of solution is related with the interaction between the limit function $k(x)$ and the spectrum of the operator $(-\Delta + \mathrm{Id})$ in $H_0^1(\Omega)$. In our case, we consider the asymptotic limit as

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