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Sensitivity analysis for generalized nonlinear parametric (A, η, m)-maximal monotone operator inclusion systems with relaxed cocoercive type operators☆

Heng-you Lan

Department of Mathematics, Sichuan University of Science & Engineering, Zigong, Sichuan 643000, PR China

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ABSTRACT

By using Lim's inequalities, Nadler's results, the new parametric resolvent operator technique associated with (A, η, m)-maximal monotone operators, in this paper, the existence theorem for a new class of generalized nonlinear parametric (A, η, m)-maximal monotone operator inclusion systems with relaxed cocoercive type operators in Hilbert spaces is analyzed and established. Our results generalize sensitivity analysis results of other recent works on strongly monotone quasi-variational inclusions, nonlinear implicit quasi-variational inclusions and nonlinear mixed quasi-variational inclusion systems in Hilbert spaces.

1. Introduction

For $i = 1, 2$, let $\mathbb{X}_i$ be real Hilbert space, Λ_i be nonempty open subset of $\mathbb{X}_i$ in which the parameter λ_i take values, and let $S, E : \mathbb{X}_1 \times \Lambda_1 \to 2^{\mathbb{X}_1}$ and $T, G : \mathbb{X}_2 \times \Lambda_2 \to 2^{\mathbb{X}_2}$ be multi-valued operators, and $g_i, p_i : \mathbb{X}_i \times \Lambda_i \to \mathbb{X}_i$, $N^1 : \mathbb{X}_1 \times \mathbb{X}_2 \times \Lambda_1 \to \mathbb{X}_1$ and $N^2 : \mathbb{X}_1 \times \mathbb{X}_2 \times \Lambda_2 \to \mathbb{X}_2$ be single-valued operators. Suppose that for $i = 1, 2$, $A_i : \mathbb{X}_i \to \mathbb{X}_i$, $\eta_i : \mathbb{X}_i \times \mathbb{X}_i \to \mathbb{X}_i$ and $M^i : \mathbb{X}_i \times \mathbb{X}_i \times \Lambda_i \to 2^{\mathbb{X}_i}$ are any nonlinear operators such that for all $(\varpi, \lambda_1) \in \mathbb{X}_1 \times \Lambda_1$, $M^1(\cdot, \varpi, \lambda_1) : \mathbb{X}_1 \to 2^{\mathbb{X}_1}$ is an (A_1, η_1, m_1)-maximal monotone operator with $(g_1 - p_1)_{\lambda_1}(\mathbb{X}_1) \cap dom(M^1(\cdot, \varpi, \lambda_1)) \neq \emptyset$ and for all $(w, \lambda_2) \in \mathbb{X}_2 \times \Lambda_2$, $M^2(\cdot, w, \lambda_2) : \mathbb{X}_2 \to 2^{\mathbb{X}_2}$ is an (A_2, η_2, m_2)-maximal monotone operator with $(g_2 - p_2)_{\lambda_2}(\mathbb{X}_2) \cap dom(M^2(\cdot, w, \lambda_2)) \neq \emptyset$, respectively, where $(g_i - p_i)_{\lambda_i}(v) = (g_i - p_i)(v(\lambda_i), \lambda_i)$ for $\lambda_i \in \Lambda_i$ and $v(\lambda_i) \in \mathbb{X}_i$. Throughout this paper, unless otherwise stated, we shall consider the following generalized nonlinear parametric (A, η, m)-maximal monotone operator inclusion systems:

For each fixed $\lambda_i \in \Lambda_i$ $(i = 1, 2)$, find $(u(\lambda_1), v(\lambda_2)) \in \mathbb{X}_1 \times \mathbb{X}_2$ such that $x(\lambda_1) \in S_{\lambda_1}(u)$, $y(\lambda_2) \in T_{\lambda_2}(v)$, $z \in E_{\lambda_1}(u)$, $\omega(\lambda_2) \in G_{\lambda_2}(v)$ and

$$\begin{cases} 0 \in N^1(u(\lambda_1), y(\lambda_2), \lambda_1) + M^1_{\lambda_1}((g_1 - p_1)_{\lambda_1}(u), z), \\ 0 \in N^2(x(\lambda_1), v(\lambda_2), \lambda_2) + M^2_{\lambda_2}((g_2 - p_2)_{\lambda_2}(v), \omega), \end{cases} \tag{1.1}$$

where $M^i_{\lambda_i}(u, v) = M^i(u(\lambda_i), v(\lambda_i), \lambda_i)$ for all $(u, v, \lambda_i) \in \mathbb{X}_i \times \mathbb{X}_i \times \Lambda_i$, $i = 1, 2$.

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E-mail addresses: hengyoulan@163.com, lanhengyou@hotmail.com.

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