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Center manifolds for impulsive equations under nonuniform hyperbolicity[☆]

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ABSTRACT

We establish the existence of smooth center manifolds under sufficiently small perturbations of an impulsive linear equation. In particular, we obtain the C^1 smoothness of the manifolds outside the jumping times. We emphasize that we consider the general case of nonautonomous equations for which the linear part has a *nonuniform* exponential trichotomy.

1. Introduction

We construct center manifolds for a large class of nonautonomous impulsive differential equations, obtained from sufficiently small perturbations of *nonuniform* exponential trichotomies. An impulsive differential equation corresponds to a smooth evolution that at certain times τ_i changes instantaneously, such as for example in a mechanical clock. There are many applications of these equations to mechanical and natural phenomena involving abrupt changes. We refer to the books [1,2] for an extensive list of references.

We consider the linear impulsive differential equation

$$x' = A(t)x, \quad t \neq \tau_i, \qquad \Delta x|_{t=\tau_i} = B_i x \tag{1}$$

in $\mathbb{R}^\ell$, and the perturbation

$$x' = A(t)x + f(t,x), \quad t \neq \tau_i, \qquad \Delta x|_{t=\tau_i} = B_i x + g_i(x). \tag{2}$$

Assuming that Eq. (1) has a *nonuniform* exponential trichotomy (see Section 2 for the definition), we establish the existence of center manifolds that are of class C^1 outside the jumping times, under sufficiently small nonlinear perturbations that are of class C^1 (up to the jumping times). We refer to [3] for a related discussion in the case of nonimpulsive differential equations.

In some sense, our proof of the center manifold theorem (for impulsive differential equations) should be considered similar to other approaches to prove a center manifold theorem. On the other hand, the presence of impulses creates a substantial technical difficulty. This requires considering from the beginning a space of discontinuous functions, with

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