The economics of forestry and a set-valued turnpike of the classical type☆

M. Ali Khan [a], Adriana Piazza [b,*]

[a] Department of Economics, The Johns Hopkins University, Baltimore, MD 21218, United States
[b] Departamento de Matemática, Universidad Técnica Federico Santa María, Avda. España 1680, Casilla 110-V, Valparaíso, Chile

A R T I C L E   I N F O

A B S T R A C T

In recent work, the authors set classical turnpike theory in the context of the economics of forestry, as developed by Mitra and Wan, and presented two far-reaching results. In this paper, we present a conceptual generalization that takes this theory and configures it around a set in the space of forest configurations rather than around the *golden-rule* forest configuration. Our set-valued analysis hinges on periodicity and yields the earlier results as corollaries under a non-interiority condition on the felicity function that shrinks the set to the point. The question that we pose, and answer, has obvious relevance to more general contexts and, in particular, to turnpike theory as developed by Samuelson, Gale, McKenzie, and their followers.

1. Introduction

In the first issue of the journal *Nonlinear Analysis*, Paul Samuelson [1] presented a periodic turnpike theorem. From the perspective of classical turnpike theory, as recently delineated in [2,3], it is exactly the type of theorem that one would expect and want. Given initial and final capital stocks over a sufficiently large but finite horizon, an optimal intertemporal resource allocation program stays arbitrarily close, most of the time, to the solution of an infinite horizon optimal intertemporal resource allocation program, even when the felicity function is subject to periodic oscillations, which is to say, even when the relevant turnpike is periodic. As explained in [4], and subsequently in [5,2], given the differing time horizons, and therefore differing programming problems that are involved, the question is more subtle than that of the asymptotic convergence of the solution of an infinite horizon variational problem, or of the continuity of its solution with respect to initial stocks. Periodicity, as in [1], only adds to the complexity of the question, though the question is now being phrased in the non-stationary register rather than the classical stationary one.

It is of interest that in the same year that he published his periodic turnpike theorem, Samuelson [6] turned his attention to the economics of forestry, and asked whether a profit maximizing firm would be led by market conditions to produce maximal long-run sustainable timber yields. The registers of the two enquiries were different: the first was in the context of a Ramseyian planner [7] concerned with long-term societal interest whereas the second concerned competition and the price theory of the firm. A decade was to pass before Mitra and Wan [8,9] reconfigured Samuelson's forestry problem