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Optimal mild solutions and weighted pseudo-almost periodic classical solutions of fractional integro-differential equations

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ABSTRACT

In this paper, we investigate a class of fractional integro-differential equations given by

$$\frac{\mathrm{d}^{\alpha}x(t)}{\mathrm{d}t^{\alpha}} = Ax(t) + f(t, x(t), Gx(t)).$$

Our main results concern the existence, uniqueness of optimal mild solutions and weighted pseudo-almost periodic classical solutions. An example is given to illustrate our results.

1. Introduction

Pseudo-almost periodic functions have many applications in several problems, for example in theory of partial differential equations, integral equations and functional differential equations. The concept of pseudo-almost periodicity was introduced by Zhang [1–3] in the early nineties. Since then, it became of great interest to many mathematicians [4–7]. The pseudo-almost periodicity is a natural generalization of the almost periodicity in the sense of Bohr. In [8], a new generalization of almost periodicity was introduced. Such a new concept is called weighted pseudo-almost periodicity. To construct the weighted pseudo-almost periodic functions, the main idea consists of enlarging the so-called ergodic component.

The fractional derivative is understood in the Riemann–Liouville sense. The origin of fractional calculus goes back to Newton and Leibnitz in the seventieth century. One observes that fractional order can be complex in viewpoint of pure mathematics and they have recently proved to be valuable in various fields of science and engineering. Indeed, one can find numerous applications in viscoelasticity, electrochemistry, electromagnetism, biology and hydrogeology. For example space-fractional diffusion equations have been used in groundwater hydrology to model the transport of passive tracers carried by fluid flow in a porous medium [9,10] or to model activator–inhibitor dynamics with anomalous diffusion [11]. For details; see the monographs [12–16] and the references therein.

Differential equations of fractional order have appeared in many branches of physics and technical sciences [17,18]. It has seen considerable development in the last decade; see [11–32] and the references therein. Recently, existence and uniqueness criteria for the various fractional differential equations were considered by Ahmad [19], Bhaskar [20], Lakshmikantham and Leela [21] et al. The nonlocal Cauchy problem was considered by Anguraj et al. [22] and the importance of nonlocal initial conditions in different fields has been discussed in [14,15] and the references therein.

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