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Asymptotic bounds of solutions for a periodic doubly degenerate parabolic equation[☆]

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ABSTRACT

This paper is concerned with a doubly degenerate parabolic equation with logistic periodic sources. We are interested in the discussion of the asymptotic behavior of solutions of the initial–boundary value problem. In this paper, we first establish the existence of non-trivial nonnegative periodic solutions by a monotonicity method. Then by using the Moser iterative method, we obtain an a priori upper bound of the nonnegative periodic solutions, by means of which we show the existence of the maximum periodic solution and asymptotic bounds of the nonnegative solutions of the initial–boundary value problem. We also prove that the support of the non-trivial nonnegative periodic solution is independent of time.

1. Introduction

In this paper, we consider the following periodic degenerate parabolic equation:

$$\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u^m|^{p-2}\nabla u^m) = u^\alpha(a - bu^\beta), \quad (x,t) \in \Omega \times \mathbb{R}^+, \tag{1.1}$$

$$u(x,t) = 0, \quad (x,t) \in \partial\Omega \times \mathbb{R}^+, \tag{1.2}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{1.3}$$

where $p \geq 2$, $m \geq 1$, $m(p-1) > 1$, $1 \leq \alpha < m(p-1)$, $\beta > 0$, Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $u_0(x)$ is a nonnegative bounded function with $u_0^m(x) \in W_0^{1,p}(\Omega)$, and $a = a(x,t)$ and $b = b(x,t)$ are continuous functions and T-periodic ($T > 0$) with respect to t.

The problem (1.1)–(1.3) can be proposed for many problems in mathematical biology and fisheries management. Reaction–diffusion equations with such a reaction term as (1.1) can be regarded as generalizations of Fisher or Kolmogorov–Petrovsky–Piskunov equations which are used to model the growth of populations (see [1–4]). Special cases included in Eq. (1.1) are the porous medium equation ($m > 1$, $p = 2$) and the p-Laplacian equation ($m = 1$ and $p > 2$). In recent years, periodic problems for these two kinds of equations have been widely studied in the literature (see [5–10] and the references therein). For $m = 1$, $p = 2$, (1.1) becomes the semilinear periodic parabolic equation and some related

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