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Multiplicity of positive radially symmetric solutions for a quasilinear biharmonic equation in the plane

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ABSTRACT

This paper is concerned with the multiplicity of positive radially symmetric solutions of the Dirichlet boundary value problem for the following two-dimensional quasilinear biharmonic equation

$$\Delta(|\Delta u|^{p-2}\Delta u) = \lambda g(x)f(u), \quad x \in B_1,$$

where B_1 is the unit ball in the plane. We apply the fixed point index theory and the upper and lower solutions method to investigate the multiplicity of positive radially symmetric solutions. We have found that there exists a threshold $\lambda^* < +\infty$, such that if $\lambda > \lambda^*$, then the problem has no positive radially symmetric solution; while if $0 < \lambda \leq \lambda^*$, then the problem admits at least one positive radially symmetric solution. Especially, there exist at least two positive radially symmetric solutions for $0 < \lambda < \lambda^*$.

1. Introduction

This paper is devoted to the study of positive radially symmetric solutions of the following boundary value problem for the two-dimensional quasilinear biharmonic equation

$$\begin{aligned} &\Delta(|\Delta u|^{p-2}\Delta u) = \lambda g(x)f(u), \quad x \in B_1,\\ &u = 0, \qquad x \in \partial B_1,\\ &\Delta u = 0, \qquad x \in \partial B_1, \end{aligned} \tag{P}$$

where $B_1 = \{x \in \mathbb{R}^N \,|\, |x| < 1\}$, $x = (x_1, x_2)$, $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$, $p > 1$ is a constant, and $\lambda > 0$ is a positive parameter. In order to discuss the radially symmetric solutions, we assume that $g(x)$ is radially symmetric, namely, $g(x) = g(|x|)$. Let $u(t) \triangleq u(|x|)$ with $t = |x|$ be a radially symmetric solution. Then direct calculations show that

$$\mathscr{L}\left(|\mathscr{L}u|^{p-2}\mathscr{L}u\right) = \lambda g(t)f(u), \quad t = |x|,\ 0 < t < 1, \tag{1.1}$$

with the boundary value condition

$$u'(0) = u(1) = \left(|\mathscr{L}u|^{p-2}\mathscr{L}u\right)'|_{t=0} = (|\mathscr{L}u|^{p-2}\mathscr{L}u)_{t=1} = 0, \tag{1.2}$$

where $\mathscr{L}$ denotes the polar form of the two-dimensional Laplacian operator Δ, i.e.

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