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Stability for steady states of Navier–Stokes–Poisson equations

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ABSTRACT

In this paper, we study the stationary solution and nonlinear stability of Navier–Stokes–Poisson equations. Using variational method, we construct steady states of the N–S–P system as minimizers of a suitably defined energy functional, then show their dynamical stability against general, i.e. not necessarily spherically symmetric perturbation.

1. Introduction

The motion of a compressible isentropic viscous fluid with self-gravitation in $\mathbb{R}^N$ ($N \geq 3$) is modeled by the Navier–Stokes–Poisson system

$$\begin{cases} \rho_t + \operatorname{div}(\rho \upsilon) = 0, \\ (\rho\upsilon)_t + \operatorname{div}(\rho\upsilon \otimes \upsilon) + \nabla P = \mu\Delta\upsilon + (\lambda + \mu)\nabla(\operatorname{div}\upsilon) - \rho\nabla\Phi, \\ \Delta\Phi = \alpha(N) g\rho, \end{cases} \tag{1.1}$$

where $\rho = \rho(x, t) \geq 0$, $\upsilon = \upsilon(x, t)$, g and Φ denote the density, velocity, gravitational constant and gravitational potential respectively, λ and μ are two viscosity coefficients satisfying

$$\mu > 0, \quad \lambda + \frac{2}{N}\mu \geq 0. \tag{1.2}$$

For $N \geq 3$,

$$\alpha(N) = N(N-2)\frac{\pi^{N/2}}{\Gamma(N/2+1)},$$

where Γ is the Gamma function. $P = P(\rho)$ is the pressure that does not depend on the temperature or specific entropy. The Poisson equation $(1.1)_3$ can be solved as

$$\Phi_\rho(x) = -g\int \frac{\rho(y)}{|x-y|^{N-2}}\mathrm{d}y. \tag{1.3}$$

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