Nonlinear Analysis 74 (2011) 2929–2940

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Global dynamics of a viral infection model with a latent period and Beddington–DeAngelis response

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ARTICLE INFO

Article history:
Received 23 September 2009
Accepted 29 December 2010

Keywords:
Virus dynamics models
Nonlinear incidence rate
Global asymptotic stability
Uniform persistence

ABSTRACT

In this paper, we study the global dynamics of a viral infection model with a latent period. The model has a nonlinear function which denotes the incidence rate of the virus infection in vivo. The basic reproduction number of the virus is identified and it is shown that the uninfected equilibrium is globally asymptotically stable if the basic reproduction number is equal to or less than unity. Moreover, the virus and infected cells eventually persist and there exists a unique infected equilibrium which is globally asymptotically stable if the basic reproduction number is greater than unity. The basic reproduction number determines the equilibrium that is globally asymptotically stable, even if there is a time delay in the infection.

1. Introduction

To develop a better understanding of a virus dynamics in vivo, mathematical models have played a significant role. By modeling the dynamics of the virus and target cells, much knowledge about the mechanism of the interactions among these components has been gained (see [1–16] and the references therein). First of all, we introduce the standard viral infection model:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}x(t) = s - \mathrm{d}x(t) - kx(t)v(t), \\ \dfrac{\mathrm{d}}{\mathrm{d}t}y(t) = kx(t)v(t) - \delta y(t), \\ \dfrac{\mathrm{d}}{\mathrm{d}t}v(t) = N\delta y(t) - \mu v(t) \end{cases} \tag{1.1}$$

where $x(t)$, $y(t)$ and $v(t)$ denote the concentration of uninfected cells, infected cells and free virus particles, respectively. It is assumed that new target cells are generated at a constant rate s and die at rate $\mathrm{d}x(t)$. Infection of target cells by free virus is assumed to occur at rate $kx(t)v(t)$ and die at rate $\delta y(t)$. The average number of virus particles produced over the lifetime of a single infected cell is N, which is also called the burst size. Hence, new virus particles are produced from infected cells at rate $N\delta y(t)$ and die at rate $\mu v(t)$. (1.1) has been used to model a virus dynamics in vivo, such as that of human immunodeficiency virus type I (HIV-1) and hepatitis B virus (HBV) (see also [2,1,14,3,13,12] and the references therein).

(1.2) always has an uninfected equilibrium $(x_0, 0, 0)$, $x_0 = \frac{s}{d}$, corresponding to the extinction of the infected cells and virus. It is also possible that an infected (internal) equilibrium $(\bar{x}_1, \bar{y}_1, \bar{z}_1)$ exists. Existence of the infected equilibrium is determined by a parameter which is called the basic reproduction number. The basic reproduction number denotes the

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0362-546X/$ – see front matter
doi:10.1016/j.na.2010.12.030