Nonlinear Analysis 74 (2011) 639–651

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Study the stability of solutions of functional differential equations via fixed points

Dingheng Pi *

Department of Mathematics, Shanghai Jiaotong University, Shanghai 200240, People's Republic of China

ARTICLE INFO

Article history:
Received 31 July 2010
Accepted 8 September 2010

MSC:
34K20
47H10

Keywords:
Stable
Asymptotically stable
Fixed points
Variable delay

ABSTRACT

In this paper, we study the stability properties of solutions of a class of functional differential equations with variable delay. By using the fixed point theory under an exponentially weighted metric, we obtain some interesting sufficient conditions ensuring that the zero solution of the equations is stable and asymptotically stable.

1. Introduction

More than 100 years ago, the world famous mathematician Lyapunov established the Lyapunov direct method to study stability problems. From then on, Lyapunov's direct method was widely used to study the stability of solutions of ordinary differential equations and functional differential equations, see e.g. [1–7] and the references therein. But the expressions of Lyapunov functional are very complicated and hard to construct.

Recently, many authors have realized that the fixed points theory can be used to study the stability of the solution. Becker and Burton considered the differential equation

$$\dot{x} = -\int_{t-r(t)}^{t} a(t,s)g(x(s))\mathrm{d}s$$

and

$$\dot{x}(t) = a(t)g(x(t-r(t))),$$

where $r(t) \geq 0$. Then obtained conditions ensuring that the zero solution was asymptotically stable by changing the supremum metric to an exponentially weighted metric, see [8]. Burton considered the equation

$$\ddot{x} + f(t,x,\dot{x})\dot{x} + b(t)g(x(t-L)) = 0,$$

where L is a positive constant. By using the fixed point theorem, he obtained sufficient conditions for each solution $x(t)$ to satisfy $(x(t), \dot{x}(t)) \to 0$, see [9].

* Tel.: +86 021 54748281.
E-mail address: pidh@sjtu.edu.cn.

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.09.018