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Some eigenvalue results for maximal monotone operators

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ABSTRACT

We study the eigenvalue problem of the form

$$0 \in Tx - \lambda Cx,$$

where X is a real reflexive Banach space with its dual X^* and $T : X \supset D(T) \to 2^{X^*}$ is a maximal monotone multi-valued operator and $C : D(T) \to X^*$ is a not necessarily continuous single-valued operator. Using the index theory for countably condensing operators, we extend some related results of Kartsatos to the countably condensing case instead of compactness of the approximant J_μ. Moreover, the solvability of the perturbed problem $0 \in Tx + Cx$ is discussed in an analogous method to the above problem.

1. Introduction and preliminaries

The theory of maximal monotone operators based on fundamental principles from functional analysis has been developed to obtain many applications to operator equations, evolution equations, and Hammerstein equations; see e.g., [1,2]. Some eigenvalue problems for maximal monotone and m-accretive operators have been extensively investigated in several ways of approach; see [3–8]. More generally, implicit eigenvalue problems were considered in [6,7]. The study was mainly based on degree theories for various classes of operators and the method of regularization by means of the duality map was used in most of the results. For instance, Browder's degree theory [9] for operators of type (S_+) applies in [7].

Let X be a real reflexive Banach space with its dual X^*. We consider the eigenvalue problem of the form

$$0 \in Tx - \lambda Cx, \tag{E}$$

where $T : X \supset D(T) \to 2^{X^*}$ is a maximal monotone multi-valued operator and $C : D(T) \to X^*$ is a single-valued operator. When the operator C or the resolvents of the operator T are compact, it was studied in [3,7,8], by applying the Leray–Schauder degree theory. The idea is to solve the following equation

$$x = J_\mu x - \lambda \mu J^{-1} C J_\mu x,$$

where J_μ is the Brezis–Crandall–Pazy approximant introduced in [10]. When J_μ is compact, Kartsatos [5] showed the existence of positive eigenvalues for problem (E).

It is natural to consider a larger class of countably condensing operators that contains compact and condensing operators. It turned out in [11] that countability, e.g., a sequence of approximate solutions, could be sufficient for solving boundary value problems for nonlinear differential equations. In this aspect, Väth [12] established a fixed point index theory for countably condensing operators.

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