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Spectral theory for linearized p-Laplace equations

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ABSTRACT

We continue and completely set up the spectral theory initiated in Castorina et al. (2009) [5] for the linearized operator arising from $\Delta_p u + f(u) = 0$. We establish existence and variational characterization of all the eigenvalues, and by a weak Harnack inequality we deduce Hölder continuity for the corresponding eigenfunctions, this regularity being sharp. The Morse index of a positive solution can be now defined in the classical way, and we will illustrate some qualitative consequences one should expect to deduce from such information. In particular, we show that zero Morse index (or more generally, non-degenerate) solutions on the annulus are radial.

1. Introduction

Let $u \in C^{1,\alpha}(\Omega)$ be a weak solution of the problem

$$\begin{cases} -\Delta_p u = f(u) & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geqslant 2$, $\Delta_p u = \mathrm{div}(|Du|^{p-2}Du)$ is the p-Laplace operator, and f is a positive ($f(s) > 0$ for $s > 0$) locally Lipschitz continuous nonlinearity. The Hölder continuity of ∇u is in general optimal [1–3] and Eq. (1.1) is always meant in a weak sense.

The linearized operator L_u associated to (1.1) at a given solution u is defined by duality as $L_u : v \in H_0 \to L_u(v) \in H_0'$, where $L_u(v) : \varphi \in H_0 \to L_u(v, \varphi)$ and

$$L_u(v, \varphi) := \int_\Omega |\nabla u|^{p-2}(\nabla v, \nabla \varphi) + (p-2)\int_\Omega |\nabla u|^{p-4}(\nabla u, \nabla v)(\nabla u, \nabla \varphi) - \int_\Omega f'(u) v\varphi. \tag{1.2}$$

The Hilbert space H_0 will be rigorously introduced in Section 2 according to [4] and is roughly composed by functions v vanishing on the boundary so that $\int_\Omega |\nabla u|^{p-2}|\nabla v|^2 < \infty$. In this way, the operator L_u is well defined, and in [5] it is shown that the first eigenvalue of L_u

$$\mu_1 = \inf_{v \in H_0,\ v \neq 0} \frac{L_u(v, v)}{\int_\Omega v^2}$$

is simple and attained at a nonnegative first eigenfunction v_1. The study in [5] can be pushed further to set up a complete spectral theory for L_u as summarized in the following.

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