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Subelliptic estimates on compact semisimple Lie groups

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ABSTRACT

In this paper, we consider a natural subelliptic structure in semisimple, compact and connected Lie groups, and estimate the constant in the so-called subelliptic Friedrichs–Knapp–Stein inequality, which has implications in the regularity theory of p-energy minimizers.

1. Introduction

In this paper, we propose to obtain the best constant for the subelliptic Friedrichs–Knapp–Stein inequality:

$$\|\mathfrak{X}^2 f\|_{L^2(G)} \leq C\|\Delta_{\mathfrak{X}} f\|_{L^2(G)}, \tag{1.1}$$

where the subelliptic structure is generated by an orthonormal basis $\mathfrak{X}$ of the orthogonal complement of the Cartan subalgebra of a semisimple, compact and connected Lie group. In inequality (1.1)

$$\|\mathfrak{X}^2 f\|_{L^2(G)} = \left(\int_G \sum_{(X,Y)\in\mathfrak{X}\times\mathfrak{X}} |XYf|^2 \mathrm{d}\mu\right)^{\frac{1}{2}}$$

is the norm of the matrix of second order horizontal derivatives and

$$\Delta_{\mathfrak{X}} f = \sum_{X\in\mathfrak{X}} XXf$$

is the subelliptic Laplacian. A look at the generators of the Lie algebra of SO(n) for $n \geq 4$ (see (3.5)) shows that a semisimple compact Lie group can be endowed with a variety of subelliptic structures, and thus, the estimates of C can vary. Once our method becomes clear, we will be able to make further comments on the advantages and disadvantages of a certain choice. We will consider a subelliptic structure that naturally appears as the result of the root space decomposition associated with a Cartan subalgebra. This is a generalization of the subelliptic structure on SU(2) – and implicitly on SO(3) – considered in [1], which uses two out of the three Pauli matrices as generators. Moreover, the proposed subelliptic structure satisfies the assumptions of [2], and therefore, we already have some regularity results available for quasilinear subelliptic PDEs, which constitute the basis of a good application of (1.1).

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