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On the existence and asymptotic behaviour of solutions of an evolution equation and an application to the Feynman–Kac theorem

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ABSTRACT

In this paper, we study the existence of mild solutions of a semilinear evolution equation on an unbounded interval. The rapidity of the growth of those solutions is characterized. Moreover, we investigate the local and global attractivity of solutions of an equation in question and we describe their asymptotic behaviour. Our investigations are conducted in a real separable Banach space.

1. Introduction

The objective of the paper is to investigate the existence and asymptotic behaviour of mild solutions on an unbounded interval of the semilinear evolution equation of the form

$$x'(t) = Ax(t) + f(t, x(t)), \quad t \in \mathbb{R}_+ = [0, \infty), \tag{1}$$

$$x(0) = x_0 \in E, \tag{2}$$

where the operator $A : D(A) \subset E \to E$ generates a C_0- semigroup $\{G(t)\}_{t\geq 0}$ and E is a real separable Banach space.

Recently a lot of papers have appeared that deal with the same or similar equations on a bounded interval (cf. [1–17]). However, only in a few papers the problem (1)–(2) was considered on an unbounded interval [8,18]. Additionally, in assumptions concerning the semigroup $\{G(t)\}_{t\geq 0}$ or the function $f(t, x)$, rather restrictive conditions have been imposed frequently that require the compactness of $f(t, x)$ or $\{G(t)\}_{t\geq 0}$, or equicontinuity of the semigroup $\{G(t)\}_{t\geq 0}$ [1–9,11–18]. It is worthwhile mentioning that only a few papers have discussed asymptotic behaviour of solutions, mostly without the formulation of existence theorems [8,19,20].

In this paper, we present conditions guaranteeing the existence of mild solutions on an unbounded interval of (1)–(2). We dispense with assumptions on the compactness of $f(t, x)$ or $\{G(t)\}_{t\geq 0}$.

Moreover, we formulate theorems about asymptotic properties and both local and global attractivity of solutions of problem (1)–(2). The existence theorems concerning that problem will be proved with the help of the technique of measures of noncompactness in the Banach space $C(\mathbb{R}_+, p(t), E)$, which will be defined later on.

The approach applied here was introduced and developed in [21–24], for instance.

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