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Qualitative analysis of a delayed free boundary problem for tumor growth under the effect of inhibitors

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ABSTRACT

In this paper we study a delayed free boundary problem for the growth of tumors under the effect of inhibitors. The establishing of the model is based on the diffusion of nutrient and inhibitors, and mass conservation for the two processes proliferation and apoptosis. It is assumed that the process of proliferation is delayed compared to apoptosis. We mainly study the asymptotic behavior of the solution, and prove that under some assumptions, in the case where c_1 and c_2 are sufficiently small, the volume of the tumor cannot expand without limit; it will either disappear or evolve to a dormant state as $t \to \infty$.

1. Introduction

An increasing number of free boundary problems of partial differential equation (P.D.E.) models for tumor growth or therapy have been developed over the last thirty years; cf. [1–8] and references therein. These models are based on the reaction–diffusion equations and the mass conservation law. Since analysis of such models not only provides a sound theoretical basis for tumor medicine, but also enriches the understanding of P.D.E., analysis of such free boundary problems has drawn great interest, and many interesting results have been established; cf. [9–23] and references therein.

In this paper we study the following problem:

$$c_1\frac{\partial\sigma}{\partial t} = \Delta_r\sigma(r,t) - \Gamma_1\sigma(r,t), \quad 0 < r < R(t),\ t > 0, \tag{1.1}$$

$$c_2\frac{\partial\beta}{\partial t} = \Delta_r\beta(r,t) - \Gamma_2\beta(r,t), \quad 0 < r < R(t),\ t > 0, \tag{1.2}$$

$$\frac{\partial\sigma}{\partial r}(0,t) = 0, \quad \sigma(R(t),t) = \sigma_\infty,\ t > 0, \tag{1.3}$$

$$\frac{\partial\beta}{\partial r}(0,t) = 0, \quad \beta(R(t),t) = \beta_\infty,\ t > 0, \tag{1.4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{4\pi R^3(t)}{3} = 4\pi\left[\int_0^{R(t-\tau)}\mu\sigma(r,t-\tau)r^2\mathrm{d}r - \int_0^{R(t)}\mu\tilde{\sigma}r^2\mathrm{d}r - \int_0^{R(t)}\nu\beta(r,t)r^2\mathrm{d}r\right], \quad t > 0, \tag{1.5}$$

$$\sigma(r,t) = \psi(r,t), \quad 0 \le r \le R(t),\ -\tau \le t \le 0, \tag{1.6}$$

$$R(t) = \varphi(t), \quad -\tau \le t \le 0 \tag{1.7}$$

where r is the radial variable scaled by the tumor cell radius, t is the time variable scaled by the tumor cell doubling time, the variables $\sigma(r,t)$ and $\beta(r,t)$, respectively, represent the scaled nutrient and inhibitor concentrations at radius r and time

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