



Coincidence and common fixed point results in partially ordered cone metric spaces and applications to integral equations

H. Aydi^a, H.K. Nashine^b, B. Samet^c, H. Yazidi^{c,*}

^a Université de Monastir, Institut Supérieur d'Informatique de Mahdia, Route de Réjiche, Km 4, BP 35, Mahdia 5121, Tunisie

^b Department of Mathematics, Disha Institute of Management and Technology, Satya Vihar, Vidhansabha-Chandrakhuri Marg, Naradha, Mandir Hasaud, Raipur-492101 (Chhattisgarh), India

^c Université de Tunis, Ecole Supérieure des Sciences et Techniques de Tunis, 5, Avenue Taha Hussein-Tunis, B.P.:56, Bab Menara-1008, Tunisie

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ABSTRACT

In this paper, coincidence and common fixed point results are established in a partially ordered cone metric space. An application of our results obtained to prove the existence of a common solution to integral equations is presented.

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1. Introduction

Fixed point theory has fascinated many researchers since 1922 with the celebrated Banach's fixed point theorem. There exists a vast literature on the topic and this is a very active field of research at present. A self-map T on a metric space X is said to have a fixed point $x \in X$ if $Tx = x$. Theorems concerning the existence and properties of fixed points are known as fixed point theorems. Such theorems are very important tools for proving the existence and eventually the uniqueness of the solutions to various mathematical models (integral and partial differential equations, variational inequalities, ...).

Huang and Zhang [1] reintroduced the concept of a cone metric space, where every pair of elements is assigned to an element of a Banach space equipped with a cone which induces a natural partial order. In the same work, they investigated the convergence in cone metric spaces, introduced the notion of their completeness, and proved some fixed point theorems for mappings satisfying different contractive conditions on these spaces. After that, fixed point results for cone metric spaces were studied by many other authors. Refs. [2–16] are some works in this line of research.

* Corresponding author.

E-mail addresses: hassen.aydi@isima.rnu.tn (H. Aydi), hemantnashine@rediffmail.com (H.K. Nashine), bessem.samet@gmail.com (B. Samet), Habib.Yazidi@essst.rnu.tn, habib.yazidi@gmail.com (H. Yazidi).