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A singular solution with smooth initial data for a semilinear parabolic equation

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ABSTRACT

We consider the Cauchy problem for a parabolic partial differential equation with a power nonlinearity. Our concern in this paper is the existence of a singular solution with smooth initial data. By using the Haraux–Weissler equation, it is shown that there exist singular forward self-similar solutions. Using this result, we also obtain a sufficient condition for the singular solution with general initial data including smooth initial data.

1. Introduction

We consider singular solutions for the semilinear parabolic equation

$$u_t = \Delta u + u^p, \quad x \in \mathbb{R}^N, \tag{1}$$

where $p > 1$ is a parameter. It is known that for $N \geq 3$ and

$$p > p_{sg} := \frac{N}{N-2}.$$

(1) has a singular steady state $\Phi_\infty(x) \in C^\infty(\mathbb{R}^N \setminus \{\xi_0\})$ with a singular point $\xi_0 \in \mathbb{R}^N$ that is explicitly expressed as

$$\Phi_\infty(x) = L|x-\xi_0|^{-m}, \quad m = \frac{2}{p-1}, \qquad L^{p-1} = m(N-m-2).$$

Since this singular steady state is radially symmetric with respect to ξ_0, we may write Φ_∞ as a function of $r = |x-\xi_0|$. Then $\Phi_\infty = \Phi_\infty(r)$ satisfies (1) in the distribution sense, and

$$(\Phi_\infty)_{rr} + \frac{N-1}{r}(\Phi_\infty)_r + (\Phi_\infty)^p = 0, \quad r = |x-\xi_0| > 0.$$

Regarding the singular solutions of (1), the exponent

$$p_* := \frac{N+2\sqrt{N-1}}{N-4+2\sqrt{N-1}}, \quad N > 2,$$

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