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Maximal monotonicity for the sum of two subdifferential operators in L^p-spaces

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This paper is dedicated to the memory of Professor Yukio Kōmura.

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ABSTRACT

This paper is devoted to providing a sufficient condition for the maximality of the sum of subdifferential operators defined on reflexive Banach spaces and proving the maximal monotonicity in $L^p(\Omega) \times L^{p'}(\Omega)$ of the nonlinear elliptic operator $u \mapsto -\Delta_m u + \beta(u(\cdot))$ with a maximal monotone graph β.

1. Introduction

Let E and E^* be a real reflexive Banach space and its dual space, respectively, and let $\phi_1, \phi_2 : E \to (-\infty, \infty]$ be proper (i.e., $\phi_1, \phi_2 \not\equiv \infty$) lower semicontinuous convex functionals with the effective domains $D(\phi_i) := \{u \in E; \phi_i(u) < \infty\}$ for $i = 1, 2$. Then the subdifferential operator $\partial_E \phi_i : E \to 2^{E^*}$ of ϕ_i is defined by

$$\partial_E \phi_i(u) := \left\{ \xi \in E^*;\ \phi_i(v) - \phi_i(u) \geq \langle \xi, v - u \rangle_E \text{ for all } v \in D(\phi_i) \right\},$$

where $\langle \cdot, \cdot \rangle_E$ denotes the duality pairing between E and E^*, with the domain $D(\partial_E \phi_i) = \{u \in D(\phi_i); \partial_E \phi_i(u) \neq \emptyset\}$ for $i = 1, 2$. This paper provides a new sufficient condition for the maximal monotonicity of the sum $\partial_E \phi_1 + \partial_E \phi_2$ in $E \times E^*$ and an application to nonlinear elliptic operators in L^p-spaces.

This paper is motivated by the question of whether the following operator $\mathcal{M}$ is maximal monotone in $L^p(\Omega) \times L^{p'}(\Omega)$ with $p \in [2, \infty)$, $p' = p/(p-1)$ and a bounded domain Ω of $\mathbb{R}^N$:

$$\mathcal{M} : D(\mathcal{M}) \subset L^p(\Omega) \to L^{p'}(\Omega); \qquad u \mapsto -\Delta_m u + \beta(u(\cdot)), \tag{1}$$

where β is a maximal monotone graph in $\mathbb{R}$ such that $\beta(0) \ni 0$, and Δ_m is a modified Laplacian given by

$$\Delta_m u = \nabla \cdot \left(|\nabla u|^{m-2} \nabla u \right), \quad 1 < m < \infty$$

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