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Lie point symmetries and some group invariant solutions of the quasilinear equation involving the infinity Laplacian

Igor Leite Freire *, Antonio Cândido Faleiros

Centro de Matemática, Computação e Cognição, Universidade Federal do ABC - UFABC, Rua Santa Adélia, 166, Bairro Bangu, CEP 09.210-170, Santo André - SP, Brazil

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ABSTRACT

Using the classical Lie method we obtain the full Lie point symmetry group of the Aronsson equation in two independent variables. Some group invariant solutions of this equation are found and a conjecture on the Lie point symmetry group of the Aronsson equation in $\mathbb{R}^n$ is presented.

1. Introduction

Let $D \subseteq \mathbb{R}^2$ be a convex region and $u \in C^1(D) \cap C^0(\overline{D})$. For any $n \in \mathbb{N}$, let

$$I_n(u) := \left(\int_D |\nabla u|^{2n} \right)^{\frac{1}{2n}}, \tag{1}$$

where $\nabla u := (u_x, u_y)$. Supposing that u is a solution of the problem

$$\min I_n(u),$$

then u satisfies the equation

$$|\nabla u|^{2(n-2)} \left[\frac{1}{2(n-1)} |\nabla u|^2 (u_{xx} + u_{yy}) + u_x^2 u_{xx} + 2u_x u_y u_{xy} + u_y^2 u_{yy} \right] = 0. \tag{2}$$

If $\nabla u \neq 0$ and if n tends to infinity, Eq. (2) becomes

$$u_x^2 u_{xx} + 2u_x u_y u_{xy} + u_y^2 u_{yy} = 0. \tag{3}$$

* Corresponding author.
E-mail addresses: igor@ufabc.edu.br, igor.freire@ufabc.edu.br (I.L. Freire), antonio.faleiros@ufabc.edu.br (A.C. Faleiros).

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