Nonlinear Analysis 74 (2011) 5685–5696

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Multiple positive solutions to singular positone and semipositone Dirichlet-type boundary value problems of nonlinear fractional differential equations

Xiaojie Xu [a,b,*], Daqing Jiang [b], Chengjun Yuan [b,c]

[a] School of Science, China University of Petroleum (East China), Qingdao 266555, China

[b] School of Mathematics and Statistics, Northeast Normal University, Changchun 130024, China

[c] School of Mathematics and Computer, Harbin University, Harbin 150086, China

ARTICLE INFO

Article history:
Received 23 March 2011
Accepted 17 May 2011
Communicated by Ravi Agarwal

MSC:
34B15

Keywords:
Multiple positive solutions
Singular fractional differential equation
Leray–Schauder nonlinear alternative
Boundary value problem

ABSTRACT

In this paper, we establish the existence of multiple positive solutions to positone and semipositone Dirichlet-type boundary value problems of the nonlinear fractional differential equation:

$$\begin{aligned} &\mathbf{D}_{0+}^{\alpha}u(t) + f(t, u(t)) = 0, \quad 0 < t < 1, \\ &u(0) = u(1) = 0 \end{aligned}$$

by using the Leray–Schauder nonlinear alternative and a fixed-point theorem on cones, where $1 < \alpha < 2$ is a real number and $\mathbf{D}_{0+}^{\alpha}$ is the standard Riemann–Liouville derivative. Here our nonlinearity f may be singular at $u = 0$.

1. Introduction

Fractional differential equations have been of great interest recently. This is due to the intensive development of the theory of fractional calculus itself as well as its applications. Apart from diverse areas of mathematics, fractional differential equations arise in rheology, dynamical processes in selfsimilar and porous structures, fluid flows, electrical networks, viscoelasticity, chemical physics, and many other branches of science. For details, see [1–7].

In this paper, we consider the existence and multiplicity of positive solutions of the nonlinear fractional differential equation semipositone boundary value problem:

$$\begin{aligned} &\mathbf{D}_{0+}^{\alpha}u(t) + f(t, u(t)) = 0, \quad 0 < t < 1, \\ &u(0) = u(1) = 0 \end{aligned} \tag{1.1}$$

where $1 < \alpha < 2$ is a real number, $\mathbf{D}_{0+}^{\alpha}$ is the standard Riemann–Liouville derivative, and f may be singular at $u = 0$. As far as we know, the nonlinear integer order differential equation for the Dirichlet boundary value problem has been studied extensively. For details, see [8–13] and the references therein. However, only a few papers have dealt with the boundary value problem for fractional differential equations.

In [14], the authors consider the nonlinear fractional differential equation Dirichlet-type boundary value problem (1.1). They derived the corresponding Green function called the fractional Green function and obtained some properties. It is well

* Corresponding author at: School of Science, China University of Petroleum (East China), Qingdao 266555, China.
E-mail address: xuxiaojie77@sohu.com (X. Xu).

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doi:10.1016/j.na.2011.05.055