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Multiple positive solutions of a nonlinear boundary value problem involving a sign-changing weight

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ABSTRACT

In this paper, we study the multiplicity of positive solutions for the following elliptic equation:

$$\begin{cases} \Delta u - u = 0 & \text{in } \mathbb{R}^N_+, \\ \dfrac{\partial u}{\partial \nu} = \lambda a\,(x)\,|u|^{q-2}\,u + b\,(x)\,|u|^{p-2}\,u & \text{on } \partial\mathbb{R}^N_+, \end{cases}$$

where $1 < q < 2 < p < 2_*$ ($2_* = \frac{2(N-1)}{N-2}$ if $N \geq 3$, $2_* = \infty$ if $N = 2$), $\mathbb{R}^N_+ = \{(x', x_N) \in \mathbb{R}^{N-1} \times \mathbb{R} \mid x_N > 0\}$ is an upper half space in $\mathbb{R}^N$, $\lambda > 0$ and the functions a and b are satisfying some suitable conditions. Using the decomposition of the Nehari manifold, we prove that the equation has at least two positive solutions provided λ is sufficiently small.

1. Introduction

In this paper, we study the multiplicity of positive solutions for the following elliptic equation:

$$\begin{cases} \Delta u - u = 0 & \text{in } \mathbb{R}^N_+, \\ \dfrac{\partial u}{\partial \nu} = \lambda a(x)\,|u|^{q-2}\,u + b(x)\,|u|^{p-2}\,u & \text{on } \partial\mathbb{R}^N_+, \end{cases} \tag{E_λ}$$

where $1 < q < 2 < p < 2_*$ ($2_* = \frac{2(N-1)}{N-2}$ if $N \geq 3$, $2_* = \infty$ if $N = 2$), $\mathbb{R}^N_+ = \{(x', x_N) \in \mathbb{R}^{N-1} \times \mathbb{R} \mid x_N > 0\}$ is an upper half space in $\mathbb{R}^N$ and $\lambda > 0$. We assume that the functions a and b satisfy the following conditions:

(D1) $a \in L^{\frac{p}{p-q}}\left(\partial\mathbb{R}^N_+\right) \setminus \{0\}$ with $a_\pm(x) = \pm\max\{\pm a(x), 0\} \not\equiv 0$;

(D2) $b \in C\left(\partial\mathbb{R}^N_+\right)$ and there is a positive number $r_b < q$ such that

$$b(x) \geq 1 + c_0 \exp\left(-r_b\,|x|\right) \quad \text{for some } c_0 < 1 \text{ and for all } x \in \partial\mathbb{R}^N_+$$

and

$$b(x) \to 1 \quad \text{as } |x| \to \infty.$$

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