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Generalized fronts in reaction–diffusion equations with mono-stable nonlinearity

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ABSTRACT

We consider transition fronts (generalized traveling fronts) of mono-stable reaction–diffusion equations with spatially inhomogeneous nonlinearity. By constructing a cut-off function and using an approximate method, we establish the existence of transition fronts of the equation. Furthermore, we give the uniform non-degeneracy estimates of the solutions, such as a lower bound on the time derivative on some level sets, as well as an upper bound on the spatial derivative.

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1. Introduction

This paper is concerned with the following reaction–diffusion equation

$$u_t = u_{xx} + f(x, u), \quad x \in \mathbb{R},\ t \in \mathbb{R}, \tag{1.1}$$

where $f(x, u)$ is continuous in x, Lipschitz continuous in u. Moreover, f satisfies the mono-stable assumption

$$\begin{cases} f(x,0) = f(x,1) = 0, \\ f(x,u) > 0, \quad u \in (0,1), \\ f_u'(x,0) := \lim\limits_{u \to 0^+} u^{-1} f(x,u) > 0. \end{cases} \tag{1.2}$$

Assume that there exist non-negative Lipschitz functions $f_0(u)$ and $f_1(u)$, decreasing on $[1-\epsilon, 1]$ for some $0 < \epsilon < 1$, such that

$$f_0(u) \le f(x,u) \le f_1(u). \tag{1.3}$$

And they satisfy

$$\begin{cases} f_0(0) = f_0(1) = f_1(0) = f_1(1) = 0, \\ f_0(u) = 0, \quad u \in [0, \theta_0], \\ f_0(u) > 0, \quad u \in (\theta_0, 1), \theta_0 \in (0,1), \\ f_1(u) > 0, \quad u \in (0,1). \end{cases}$$

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