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Lagrangean variables in infinite dimensional spaces for a dynamic economic equilibrium problem

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ABSTRACT

This paper is focused on the study of a dynamic competitive equilibrium by using Lagrangean multipliers. This mathematical formulation allows us the improve the Walrasian model by considering the common possibility of an uncharged delayed payment in a given time (for example, by using a credit card). Firstly the economic equilibrium problem is reformulated as an evolutionary variational problem; then the Lagrangean theory in infinite dimensional spaces is applied. Thanks to the application of this theory we obtain the existence of Lagrangean multipliers, which allows us to give a computational procedure for the equilibrium solutions.

1. Introduction

The duality and Lagrangean theory provide interesting contributions, absolutely necessary for the better understanding and handling of an increasing number of equilibrium problems whether in the static case or in the evolutionary case: the traffic equilibrium problem, the financial equilibrium problem, the spatially distributed markets equilibrium problem, the oligopolistic market equilibrium problem, the mixed equilibrium problem, the pollution control problem, the vaccination problem, and the Walras equilibrium problem (see e.g. [1–10]). Each of the aforementioned problems can be transformed into a variational problem on a convex subset K of a suitable functional space, for which an impressive quantity of results in terms of existence, calculation of the solutions, stability and sensitivity analysis hold. This study is also undertaken by applying a suitable duality and Lagrangean theory in order to obtain necessary and/or sufficient optimality conditions. The Lagrangean theory is useful not only to provide a qualitative analysis of equilibria, but also to calculate the solutions.

In this paper, a dynamic competitive equilibrium for a Walrasian pure exchange economy has been considered. This problem was introduced by Walras in [11]. The equilibrium model is built through the maximization of a utility function from which a nonpositive excess demand function is derived. Still now the mathematical study of the Walras equilibrium represents an active and interesting research topic (see e.g. [12–15,8,16–18]). In this paper, we introduce a continuum model of a competitive equilibrium for a pure exchange economy in the finite interval $[0, T]$ and we give a dynamic formulation of the equilibrium conditions. In this evolutionary market the aim of each agent is not to maximize his utility at the fixed instant t, but to maximize it globally in the whole period of time. During the interval of time $[0, T]$ each agent trades his own commodities with the other agents taking into account his own budget set: the amount that each agent pays for acquiring the goods in the whole period $[0, T]$ is at most the amount that each agent receives, during the period $[0, T]$, as his initial endowment. Then, mathematically the equilibrium is reformulated in terms of a maximization problem of an integral utility function on a feasible set, in which the constraint is given in an integral form and in terms of a nonpositive excess demand function. First, we prove that the equilibrium is a solution to a suitable evolutionary quasi-variational inequality which is set in the Lebesgue space $L^2([0, T], \mathbb{R})$. Our main result in this paper is to obtain the Lagrangean variables relative to the

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