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Multiple periodic solutions of superlinear ordinary differential equations with a parameter☆

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ABSTRACT

We study the existence of multiple 2π-periodic solutions of ordinary differential equation $-\ddot{x} = \lambda x + f(t, x)$ with superlinear terms via homological linking and Morse theory.

1. Introduction

In this paper we are concerned with the existence of 2π-periodic solutions of the ordinary differential equation

$$\begin{cases} -\ddot{x} = \lambda x + f(t, x), \\ x(0) = x(2\pi), \qquad \dot{x}(0) = \dot{x}(2\pi), \end{cases} \tag{P_λ}$$

where $\lambda \in \mathbb{R}$ is a parameter and the nonlinear term f satisfies the following conditions:

(f_1) $f \in C^1(\mathbb{R} \times \mathbb{R}, \mathbb{R})$ is 2π-periodic in t.
(f_2) $f(t, 0) = 0, f'_x(t, 0) = 0$.
(f_3) There exist $\bar{r} > 0$ and $\mu > 2$ such that

$$0 \leqslant \mu F(t, x) := \mu \int_0^x f(t, s)\, ds \leqslant f(t, x)x, \qquad |x| \geqslant \bar{r}, \quad t \in [0, 2\pi].$$

As $f(t, 0) \equiv 0$, $(P)_\lambda$ has a trivial solution $x \equiv 0$ for any parameter $\lambda \in \mathbb{R}$. Our interest is in the multiplicity of nontrivial 2π-periodic solutions of $(P)_\lambda$ for a certain range of the parameter. We will prove that $(P)_\lambda$ has at least three nontrivial 2π-periodic solutions when *the parameter* λ *is close to any a fixed eigenvalue* of the linear periodic boundary value problem

$$\begin{cases} -\ddot{x} = \mu x, \\ x(0) = x(2\pi), \qquad \dot{x}(0) = \dot{x}(2\pi) \end{cases} \tag{L_0}$$

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